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Compactification with flux on K3 and tori

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Abstract: We study compactifications of type-IIB string theory on a $K3 \times T^2/Z_2$ orientifold in the presence of RR and NS flux. We find the most general supersymmetry preserving, Poincare invariant, vacua in this model. All the complex structure moduli and some of the Kähler moduli are stabilised in these vacua. We obtain in an explicit fashion the restrictions imposed by supersymmetry on the flux, and the values of the fixed moduli. Some T-duals and Heterotic duals are also discussed, these are non-Calabi-Yau spaces. A superpotential is constructed describing these duals. Finally, we consider some susy breaking vacua where the flux gives rise to large extra dimensions with vanishing tree-level cosmological constant.

Keywords: Superstrings and Heterotic Strings, Superstring Vacua.

Contents

1. Introduction

Compactifications of String Theory in the presence of flux have attracted considerable attention lately [1]–[27]. These compactifications are of interest from several different points of view. Phenomenologically they are attractive because turning on flux typically leads to fewer moduli, and also because flux leads to warping which is of interest in the Randall Sundrum scenario, [28, 14]. Cosmologically they are worth studying because the resulting potential in moduli space could lead to interesting dynamics. From a more theoretical view point these models enlarge the class of susy preserving vacua in string theory and one hopes this will improve our understanding of $\mathcal{N}=1$ string theory.

In this paper, we will study type-IIB string theory in the presence of flux. General considerations pertaining to this case were discussed in [6]. Subsequently, a concrete example was studied in [9] involving flux compactifications of IIB on an T^6/Z_2 orientifold. In many ways this paper can be thought of as an continuation of the investigation begun in [9]. Here we study the next simplest case, where the compactification is on a $K3\times T^2/Z_2$ orientifold. The purpose behind these investigations is two fold. Qualitatively, one would like to gain some appreciation for how easy it is to obtain stable supersymmetric vacua after flux is turned on. This is important, in view of the discussion in [6] which shows that supersymmetry is generically broken once flux is turned on, and also bearing in mind that previous attempts to turn on flux have usually lead to unstable vacua with runaway behaviour [29]–[31]. Quantitatively, one would like to know how much information can be obtained about the susy preserving vacua, whether the required non-genericity of the flux can be spelt out in an explicit manner, and whether the location of the minimum in moduli space can be determined and the resulting masses of moduli be obtained.

We will carry out this investigation in detail in this paper. We solve the supersymmetry conditions explicitly to obtain the general susy preserving vacua in this model. The main point worth emphasising about our analysis is that we obtain our result without having to explicitly parametrise the moduli space of complex and Kähler deformations on $K3$. Such a parametrisation would both be inelegant and impractical. Instead, by using an important theorem, called Torelli's theorem [32], which pertains to the complex structure of $K3$, we are able to map the problem of finding susy preserving vacua into a question of Linear Algebra in the second cohomology group $H^2(K3,\mathbb{R})$ of $K3$. This question turns out to be easy to answer and yields the general susy preserving vacua mentioned above.

Our analysis allows us to state the required conditions on the flux for a susy preserving vacuum in a fairly explicit manner. We find that, qualitatively speaking, these conditions are easy to meet, so that several susy preserving vacua exist. The complex structure moduli are generically completely frozen in these vacua and some but not all of the Kähler moduli are also fixed. It is quite straightforward to determine in a quantitatively precise manner the location of the vacua in moduli space.

This paper is organised as follows. Section 2 discusses some important preliminary material. Section 3 which contains some of the key results of the paper, describes the general strategy mentioned above for finding supersymmetry preserving vacua. This general discussion is illustrated by examples in section 4 where two cases are discussed in some length.

The case of $\mathcal{N} = 2$ supersymmetry does not fall within the general discussion of section 3 and is analysed in section 5 with an example. The resulting moduli spaces of complex and Kähler deformations are also determined.

In section 6 we construct an interesting one-parameter family of susy breaking minima of the potential. These arise as follows: one can find a one parameter family of flux such that the resulting vacua all meet the $(2,1)$ condition but not the primitivity condition imposed by supersymmetry. This results in supersymmetry breaking minima of the tree-level potential with vanishing cosmological constant. Large values of flux in this family lead to the complex structure being stabilised at extreme values.

Various dual descriptions of the $K3 \times T^2/Z_2$ model are discussed in in section 7. One and two T-dualities give rise to type $I^{'}$ and type-I descriptions. The latter in turn, after S-duality, gives rise to models in heterotic string theory. The resulting compactifications are non-Calabi-Yau spaces in general. An explicit superpotential, which is valid quite generally, is constructed in these dual descriptions. It depends on various fluxes and certain twists in the geometry.

Our methods can be used to provide a general solution for flux compactifications on T^6/Z_2 considered in [9] as well (building on an approach discussed in that paper). This is discussed in section 8 briefly. The susy breaking family studied in section 5 has an analogue for the T^6/Z_2 case. Although the analysis cannot be strictly controlled once susy is broken, we find that the "large" values of flux result in two large extra dimensions, along the lines of [33, 34].

Finally, some details are discussed in the appendices A, B and C.

Let us end by commenting that the $K3 \times T^2/Z_2$ model with flux, studied in this paper, has also been analysed in [22]. Various features of the compactification were deduced in that paper by considering an M-theory lift, and also the heterotic dual was discussed in some detail [24] . While this manuscript was in preparation we became aware of [35], which discusses various aspects of Heterotic compactifications with flux in some depth. We thank the authors for discussion and for keeping us informed of their results prior to publication.

2. Background

Some background material for our investigation is discussed in this section. section 2.1 discusses some of the essential features of IIB compactifications with flux, section 2.2 gives more details relevant to the $K3 \times T^2/Z_2$ case. section 2.3 discusses some basic facts about $K3$ that will be relevant in the discussion below. Finally section 2.4 briefly discusses the lifting of open string moduli.

JHEP03(2003)028

2.1 IIB compactifications with flux

Our starting point is a compactification of IIB string theory on a $K3 \times T^2/Z_2$ orientifold. The orientifold Z_2 group is given by $\{1, \Omega(-1)^{F_L} R\}$ where R is a reflection which inverts the two coordinates of the T^2 and Ω and F_L stand for orientation reversal in the world sheet theory and fermion number in the left moving sector respectively. This model is T-dual to type-I theory on $K3 \times T^2$ which in turn is S-dual to Heterotic on $K3 \times T^2$.

The main aim of this paper is to study supersymmetric compactifications after turning on various NS-NS and R-R fluxes in this background. The general analysis of such flux compactifications was carried out in [6]. Let us summarise some of their main results here.

Turning on flux alters the metric of the internal space by an overall warp factor. For the case at hand this means upto a conformal factor the internal space is still $K3 \times T^2/Z_2$.

Define

$$G_3 = F_3 - \phi \mathcal{H}_3 \,, \tag{2.1}$$

where $F_3 = dC_2, \mathcal{H}_3 = d\mathcal{B}_2$ denote the RR and NS three forms and $\phi = a + i/g_s$ denotes the axion-dilaton. (Our notation closely follows that of [6], see also [9]). $N = 1$ supersymmetry requires that G_3 is of type $(2,1)$ with respect to the complex structure of $K3 \times T^2/Z_2$, i.e. it has index structure $[G_3]_{ij\bar{k}}$ where i, j denote holomorphic indices and $\bar{k}$ an anti holomorphic index. Furthermore for supersymmetry one needs G_3 to be primitive, that is

$$J \wedge G_3 = 0 \,, \tag{2.2}$$

where J denotes the Kähler two-form on $K3 \times T^2/Z_2$.

One of the main motivations for studying compactifications with flux is moduli stabilisation. The requirement that G_3 is $(2,1)$ typically completely fixes the complex structure for the compactification, we will see this in the analysis below for the example at hand. The primitivity condition (2.2) is automatically met if the compactification is a (conformally) Calabi-Yau threefold (CY_3), since there are no non-trivial 5-forms on a CY_3. For $K3 \times T^2$ in contrast this condition is not automatic and does impose some restriction on the Kähler moduli. An important Kähler modulus which is left unfixed is the overall volume. Since the primitivity condition is unchanged by an overall rescaling $J \to \lambda J$, it does not lift this modulus.

One comment about non-susy vacua is also making at this stage. The low-energy supergravity obtained after turning on flux is of the no-scale type. The flux gives rise to a potential for the moduli with minima at zero energy. The requirements imposed by minimising the potential are less restrictive than those imposed by supersymmetry. To minimise the potential G_3 can have components both of $(2,1)$ type and $(0,3)$ type and need not be primitive. We will have more to say about some non-susy vacua where the primitivity condition is not met in section 6.

2.2 The $K3 \times T^2/Z_2$ case in more detail

It is worth commenting more on some details relevant to the $K3 \times T^2/Z_2$ compactification we will study in this paper.

JHEP03(2003)028

The first comment relates to tadpole cancellation for various RR fields. Consider first the 7-brane tadpole. The Z_2 orientifold symmetry has 4 fixed points on the T^2, an $O7$-plane is located at each of these fixed points. To cancel the resulting 7−brane charge 16 D7-branes need to be added.

Next consider the 3-brane tadpole [36] . Both the $O7$-planes and the $D7$-branes wrap the $K3$ (besides filling spacetime). As a result, it turns out, 2 units of D3 brane charge are induced on the world volume of each $O7$-plane and one units of $D3$ brane charge in each D7-brane,[1] giving rise to a total of 24 units of three brane charge. This charge needs to be cancelled by adding $D3$-branes and flux. The relevant formula for tadpole cancellation then takes the form:

$$\frac{1}{2}N_{\text{flux}} + N_{D3} = 24\,, \tag{2.3}$$

where

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int_{K3\times T^2} \mathcal{H}_3 \wedge F_3\,. \tag{2.4}$$

Note in particular that with our normalisations the integral in (2.4) is over the covering manifold, before identification under the orientifold Z_2 symmetry. Actually, (2.3) is not the most general expression for the tadpole condition, instantons excited in the world volume of the 7-branes will give rise to 3-brane charge in general. In this paper we will only deal with the case where these instantons are not excited.

The second comment pertains to the number of moduli from the closed string sector present in the model before flux is turned on. In the closed string sector the fields g_{MN}, C_4, C_0, ϕ are even under the action of $\Omega(-1)^{F_L}$, while $\mathcal{B}_2, C_2$ are odd. This means one gets 4 gauge bosons from $\mathcal{B}_2$ and C_2. In addition the metric, and C_4 give rise to 61 and 25 scalars respectively, yielding a total of 86 scalars. The resulting compactification has $\mathcal{N}=2$ supersymmetry (before flux is turned on) with the gravity multiplet, three vector multiplets and twenty hypermultiplets. In addition, while we do not provide a precise count here, there are moduli that arise from the open string sector. These include moduli due to exciting the gauge fields on the 7 branes, the location of the 7-branes on the T^2 and the locations of the $D3$ branes in the $K3 \times T^2/Z_2$.

Finally, we discuss the quantisation conditions which must be met by the $\mathcal{H}_3$ and F_3 flux. Due to the discrete identification in the compactification this condition is a bit subtle. The orientifold $K3 \times T^2/Z_2$ has additional 'half' three-cycles not present in the covering manifold $K3 \times T^2$. To satisfy the quantisation condition on these three-cycles one requires that

$$\frac{1}{(2\pi)^2\alpha'}\int_\gamma F_3 = 2\mathbb{Z}\,, \qquad \frac{1}{(2\pi)^2\alpha'}\int_\gamma \mathcal{H}_3 = 2\mathbb{Z}\,, \tag{2.5}$$

where γ is an arbitrary class of $H_3(K3\times T^2, \mathbb{Z})$. Other possibilities which include turning on exotic flux at the O planes were discussed in [10] but will not be explored further here.

In the discussion below we will explicitly parametrise the fluxes as follows. Choose coordinate x, y for the T^2, with $0 \le x, y \le 1$. The flux which is turned on must be

[1]The simplest way to see this is the following: in F-theory there are no $O7$ planes. Instead there are 24 (p,q) 7-branes each of which acquires one unit of 3-brane charge on wrapping $K3$.

consistent with the orientifold Z_2 symmetry. Since $\mathcal{B}_2$ and C_2 are odd under $\Omega(-1)^{F_L}$ this means the allowed F_3 and $\mathcal{H}_3$ must have two legs along the $K3$ and one along the T^2. That is

$$\frac{1}{(2\pi)^2\alpha'}F_3 = \alpha_x \wedge dx + \alpha_y \wedge dy \tag{2.6}$$

$$\frac{1}{(2\pi)^2\alpha'}\mathcal{H}_3 = \beta_x \wedge dx + \beta_y \wedge dy\,, \tag{2.7}$$

where $\alpha_x, \alpha_y, \beta_x, \beta_y \in H^2(K3,\mathbb{Z})$. If e_i is a basis of $H^2(K3,\mathbb{Z})$ and $\alpha_x = \alpha_x^i e_i$ we have taking into account the quantisation condition that α_x^i is even integer, similarly for $\alpha_y, \beta_x, \beta_y$. Finally, we note that with our choice of orientation, as explained in appendix A, $N_{\rm flux}$ (2.4), takes the form

$$N_{\rm flux} = \int (\beta_x \wedge \alpha_y \wedge dx \wedge dy + \beta_y \wedge \alpha_x \wedge dy \wedge dx) = (-\beta_x \cdot \alpha_y + \beta_y \cdot \alpha_x)\,. \tag{2.8}$$

2.3 Some essential facts about $K3$

$K3$ is the two (complex) dimensional Calabi-Yau manifold. It is Kähler and has SU(2) holonomy. For an excellent review, see [32] .

$H^2(K3,\mathbb{R})$ is 22 dimensional. An inner product can be defined in $H^2(K3,\mathbb{R})$ as follows. If $e_i, e_j \in H^2(K3,\mathbb{R})$

$$(e_i, e_j) \equiv \int_{K3} e_i \wedge e_j\,. \tag{2.9}$$

This inner product matrix can be shown to have signature (3,19). $H^2(K3,\mathbb{R})$ with this metric can be naturally embedded in $\mathbb{R}^{3,19}$.

$H^2(K3,\mathbb{Z})$, can be thought of as a lattice. It is known to be even and self dual. These two conditions are highly restrictive. In a particular basis e_i (discussed in appendix A) the inner product matrix (2.9), can be shown to have the form, eq. A.3 (appendix A) consistent with these restrictions. We will refer to the lattice, together with this inner product, as $\Gamma^{3,19}$, below.

The moduli space of complex structures on $K3$ is particularly relevant for this paper since many directions lifted by flux lie in this moduli space. Torelli's theorem is important in this context. It says that, upto discrete identifications, the moduli space of complex structures on $K3$ is given by the space of possible periods of the holomorphic two-form Ω.

Decomposing Ω into its real and imaginary parts we have

$$\Omega = x + iy \tag{2.10}$$

where $x, y \in H^2(K3,\mathbb{R})$. Ω satisfies two conditions

$$\int \Omega \wedge \Omega = x\cdot x - y\cdot y + 2ix\cdot y = 0\,, \tag{2.11}$$

and

$$\int \Omega \wedge \bar{\Omega} = x\cdot x + y\cdot y > 0\,. \tag{2.12}$$

From (2.11) we see that x, y are orthogonal and from (2.12) that both x, y are space-like. So x, y span a space-like two-plane in $H^2(K3, \mathbb{R})$ or equivalently $\mathbb{R}^{3,19}$. Changing the orientation of the two-plane corresponds to taking $\Omega \leftrightarrow \bar{\Omega}$. The space of possible periods is then the space of oriented two-planes in $\mathbb{R}^{3,19}$. This space is called the grassmanian,

$$G = O(3,19)^+/(O(2) \times O(1,19))^+ \,. \tag{2.13}$$

It is twenty (complex) dimensional.

Next, let us consider the Kähler two-form, $\tilde{J}$ on $K3$. Since $K3$ is Kähler, $\tilde{J} \in H^2(K3, \mathbb{R})$. In addition it satisfies two conditions:

$$\int \tilde{J} \wedge \Omega = 0 \,, \tag{2.14}$$

and

$$\int \tilde{J} \wedge \tilde{J} > 0 \,. \tag{2.15}$$

With respect to the inner product, (2.9), the first condition tells us that $\tilde{J}$ is orthogonal to Ω, while the second says that it is a space-like two-form.

Putting this together with what we learnt above, we see that the choice of a complex structure and Kähler two-form on $K3$ specifies an oriented space-like three-plane Σ in $H^2(K3, \mathbb{R}) \in \mathbb{R}^{3,19}$. The Einstein metric is completely specified once this choice is made. It is then easy to see that the moduli space of Einstein metrics on $K3$, upto discrete identifications, is of the form

$$M_E \simeq O^+(3,19)/(\mathrm{SO}(3) \times O(19)) \times R_+ \,. \tag{2.16}$$

This is 58 real dimensional.[2]

Finally, we will work in the supergravity approximation in this paper. With that in mind, it will be useful to know in the following discussion when the the moduli are stabilised at a point away from an orbifold singularity of $K3$. Theorem 4 in [32] tells us that this condition is met if the space-like three plane Σ is not orthogonal to any Lattice Vector[3] of $\Gamma^{3,19}$.

2.4 Open string moduli

Most of this paper deals with closed string moduli. Before going on though it is worth briefly commenting on open string moduli. These moduli are of two kinds. If D3-branes are present, their location in the $K3 \times T^2/Z_2$ space gives rise to moduli. Turning on flux does not freeze these fields as long as the conditions for supersymmetry are met, this can be easily seen from the supersymmetry analysis, [37].

The second kind of open string moduli are the location of the D7- branes in the transverse T^2 directions. There are 16 such branes present. In the analysis below we take them to be symmetrically distributed so that each O7 plane has 4 D7 branes on it. As a result the dilaton will be constant (except for the singularities at the D7/O7 planes). We then consistently seek solutions which meet the conditions for supersymmetry, discussed in section 2.1.

[2] Different choices of an oriented two-plane in the chosen three-plane give rise to different complex structures consistent with the same Einstein metric. This is a reflection of the underlying Hyper Kähler geometry.

[3] In terms of the basis e_i, a Lattice Vector is of form, $n^i e_i, n^i \in \mathbb{Z}$.

Perturbing around such a solution, one can show that generically the 7-brane moduli are lifted. There are two ways to see this. From the point of view of ten-dimensional IIB theory, this happens because of the restrictive nature of the $(2,1)$ condition that G_3 must satisfy. Note that this condition must be met at every point in the compactification. Once the D7-branes move away from the O7-planes, the dilaton varies and the $(2,1)$ condition will not be met generically. From the F-theory viewpoint this can be understood similarly as a consequence of the restrictive nature of the $(2,2)$ condition that G_4 must satisfy. It is easy to see that any susy solution with constant dilaton in the IIB theory lifts to a susy preserving solution of F-theory with G_4 being of type $(2,2)$ and primitive. Now the $(2,2)$ condition imposes strong constraints on the complex structure moduli and in fact over determines them. As a result, perturbing around a given solution we generically do not expect any complex structure moduli to survive. From the IIB perspective this means all the D7-brane moduli as well as the dilaton and the complex structure of the T^2 should be fixed.

Let us end by commenting that some of these issues are discussed in more detail, from the perspective of the Heterotic theory, in [35].

3. Supersymmetric vacua

In the following two sections we solve the conditions imposed by supersymmetry and find the general supersymmetric vacua for flux compactifications on $K3 \times T^2/Z_2$. Since the discussion is quite technical it is helpful to first summarise the key ideas.

The conditions imposed by supersymmetry are well known. The flux, G_3, should be of type $(2,1)$ and it should satisfy the requirement of primitivity.

The main challenge is to explicitly implement these conditions and find susy preserving vacua. The $(2,1)$ condition in particular is very restrictive, and, as we shall see below, cannot be met for generic fluxes. In finding the general supersymmetric solutions we will determine both the allowed flux and the resulting susy vacua. A brute force approach, relying on an explicit parametrisation of moduli space, is not practical for this purpose. For example, the complex structure moduli space of $K3$ is 20 dimensional and not easy to explicitly parametrise.

The key to making progress is Torelli's theorem for $K3$, which we discussed in section 2.3 above. This theorem allows us to restate the search for susy solutions as a problem in Linear Algebra in $H^2(K3,\mathbb{R})$. Seeking a complex structure in which the flux is of type $(2,1)$ translates to searching for an appropriate space-like two-plane in $H^2(K3,\mathbb{R})$. The restrictive nature of the $(2,1)$ condition, which we mentioned above, can now be turned into an advantage. The flux defines a four dimensional subspace of $H^2(K3,\mathbb{R})$, which we denote as V_{flux}. Susy requires that the two-plane must lie in V_{flux}. This is itself a big simplification since it narrows the search from the 22 dimensional space, $H^2(K3,\mathbb{R})$, to a four dimensional one. But in fact one can do even better. One finds that the $(2,1)$ condition determines the two-plane completely in terms of the dilaton-axion and the complex structure modulus of the T^2. The remaining conditions then determine these two moduli and provide some of the required conditions on the flux.

Turning next to the requirement of primitivity, which is a condition on the Kähler two-form, one finds it maps to the search for a space-like vector in $H^2(K3, \mathbb{R})$ which is orthogonal to V_{flux}. Unlike in the case of the complex structure, this is not a very restrictive condition. It does impose some additional conditions on the flux, but once these conditions are met, many solutions exist in which some but not all Kähler moduli are frozen.

In this way we determine the most general susy preserving solutions for flux compactifications on $K3 \times T^2/Z_2$.

The rest of this section will present the analysis sketched out above in more detail. In section 3.1 we discuss the $(2, 1)$ condition. We show that it determines the complex structure of $K3$ in terms of the dilaton-axion and the complex structure of the T^2. We also find the constraints it imposes on these two moduli and on the flux. In section 3.2 we solve these constraints explicitly and determine the complex structure of $K3$, T^2 and the dilaton-axion. We also make the constraints on the flux explicit. In section 3.3 we discuss the primitivity condition. We determine the restrictions it imposes on the Kähler moduli and on the flux. Finally, we summarise the results of this section in section 3.4 stating the resulting values of the complex structure moduli, the restrictions on the Kähler moduli, and the conditions imposed on the flux for susy preserving vacua.

3.1 The $(2, 1)$ condition

We start by considering the restrictions imposed by the condition that G_3 is of type $(2, 1)$.

One way to formulate these conditions is to construct the superpotential [1, 6],

$$W = \int G_3 \wedge \Omega_3 \tag{3.1}$$

which is a function of all the complex structure moduli and also of the axion-dilaton. One can show then that imposing the conditions:

$$W = \partial_i W = 0\,, \tag{3.2}$$

where i denotes any complex structure modulus or the dilaton field, ensures that G_3 is purely of type $(2, 1)$. Notice that we have one more condition than variables in (3.2) . Two conclusions follow from this. First, as was mentioned above, we see that for generic fluxes there are no susy preserving minima. Second, we learn that in the class of fluxes which preserve susy generically all complex structure moduli are stabilised. In finding susy solutions we will determine below what conditions the flux must satisfy along with the resulting values for the complex structure moduli and the dilaton.

A straightforward way to proceed, as followed in [9], is to explicitly parametrise the complex structure moduli space, determine W, and then search for solutions. For the case at hand, this is not very practical. The main complication is the complex structure moduli space of $K3$. An explicit parametrisation of the twenty dimensional grassmanian is possible, but the resulting expressions are quite unwieldy.

The crucial idea which allows us to make progress is Torelli's theorem, as was mentioned above. We are seeking a complex structure, with respect to which G_3 is of type $(2, 1)$. Torelli's theorem states that the complex structure is specified by a space-like two plane in $H^2(K3,\mathbb{R})$. We can think of this two-plane as determining the holomorphic two-form Ω and from it the complex structure. With this in mind we recast the search for the required complex structure in terms of conditions on the two-plane. Fortunately, as we see below, these conditions are simple to state and restrictive enough to determine the two-plane without requiring an explicit parametrisation of the complex structure moduli space of $K3$.

We begin by choosing the complex structure modulus on the T^2 to be τ, so that we can define complex coordinates on it of the form,

$$z = x + \tau y\,, \qquad \bar{z} = x + \bar{\tau} y\,. \tag{3.3}$$

In (2.6), (2.7), we defined four vectors, $\{\alpha_x, \alpha_y, \beta_x, \beta_y\} \in H^2(K3,\mathbb{Z}) \subset H^2(K3,\mathbb{R})$ which specify the three-form flux completely. We will refer to these as the flux vectors below. Together they define a subspace of $H^2(K3,\mathbb{R})$ which we call V_{flux}. G_3 can then be written as

$$G_3 = n_x dx + n_y dy\,, \tag{3.4}$$

where the 2-forms n_x, n_y are given by

$$n_x \equiv \alpha_x - \phi\beta_x\ , n_y \equiv \alpha_y - \phi\beta_y\,. \tag{3.5}$$

In terms of the complex coordinates on the T^2, (3.3), we can express G_3 as

$$G_3 = \frac{1}{\bar{\tau} - \tau}\left(G_z dz + G_{\bar{z}} d\bar{z}\right) \tag{3.6}$$

with

$$G_z = (n_x \bar{\tau} - n_y) \tag{3.7}$$

and

$$G_{\bar{z}} = -(n_x \tau - n_y)\,. \tag{3.8}$$

For G_3 to be of $(2,1)$-type the two form $G_{\bar{z}}$ in $K3$ must be of type $(2,0)$. Since the Holomorphic two-form is unique we learn that

$$G_{\bar{z}} = c\,\Omega\,, \tag{3.9}$$

where c is a constant. We must emphasise that (3.9) determines the complex structure of $K3$ completely in terms of the flux, and the moduli, ϕ, τ (which enter in (3.8)).[4] Eq. (3.9), will play a central role in the discussion below.

Now we can use the remaining conditions imposed by susy and consistency to determine the remaining two moduli, the dilaton-axion and τ, and also obtain the required conditions which should be satisfied by the non-generic flux for a susy condition to exist.

From (2.11) we see that the following condition must hold:

$$G_{\bar{z}} \cdot G_{\bar{z}} = 0\,. \tag{3.10}$$

[4]We will assume in this section that $G_{\bar{z}}$ is non-zero, so that the constant c in (3.9) is non-zero. The case where $G_{\bar{z}} = 0$ is dealt with separately in section 5.

Two other conditions arise as follows. For G_3 to be of type $(2,1)$ we see from (3.7), that G_z must be of type $(1,1)$. This means

$$\Omega \cdot G_z = 0\,, \tag{3.11}$$

and

$$\Omega \cdot \bar{G}_z = 0\,, \tag{3.12}$$

which can be reexpresed as

$$G_{\bar{z}} \cdot G_z = 0 \tag{3.13}$$

and

$$G_{\bar{z}} \cdot \bar{G}_z = 0\,. \tag{3.14}$$

One final condition arises from the requirement that the two-plane defining Ω is space-like, (2.12). From (3.9), it takes the form:

$$G_{\bar{z}} \cdot \bar{G}_{\bar{z}} > 0\,. \tag{3.15}$$

We see that, (3.10), (3.13), and, (3.14), are three polynomial equations in two variables, ϕ and τ. Generically they will not have a solution. This was expected from our discussion of the superpotential at the beginning of this section. Two of these equations can be used to solve for ϕ, and τ. The third equation then gives two real conditions on the flux. Finally, we also need to ensure that the flux meets the inequality (3.15).

3.2 Solving the equations

In this section we discuss in more detail how to explicitly solve the three equations, (3.14), (3.10), (3.15), which were obtained above from the $(2,1)$ requirement.

These equations can be written as

$$(\bar{n}_x\tau - \bar{n}_y) \cdot (n_x\tau - n_y) \;=\; 0 \tag{3.16}$$

$$(n_x\bar{\tau} - n_y) \cdot (n_x\tau - n_y) \;=\; 0 \tag{3.17}$$

$$(n_x\tau - n_y) \cdot (n_x\tau - n_y) \;=\; 0\,. \tag{3.18}$$

We will in particular be interested in non-singular solutions for which $\mathrm{Im}\phi, \mathrm{Im}\tau \neq 0$. Since, $\mathrm{Im}\tau \neq 0$, (3.17), (3.18), give

$$n_x \cdot (n_x\tau - n_y) = 0\,, \tag{3.19}$$

and

$$n_y \cdot (n_x\tau - n_y) = 0\,. \tag{3.20}$$

Similarly, since, $Im\phi \neq 0$ (3.16), (3.18), give

$$(\alpha_x\tau - \alpha_y) \cdot (n_x\tau - n_y) = 0\,. \tag{3.21}$$

Using (3.19) we can eliminate τ from the remaining two equations to get,

$$(n_x \cdot n_x)(n_y \cdot n_y) - (n_x \cdot n_y)^2 \;=\; 0 \tag{3.22}$$

$$(\alpha_x \cdot n_x)(n_x \cdot n_y)^2 - (n_x \cdot n_x)(n_x \cdot n_y)\,(\alpha_x \cdot n_y + \alpha_y \cdot n_x) + (\alpha_y \cdot n_y)(n_x \cdot n_x)^2 \;=\; 0\,. \tag{3.23}$$

Using the expressions for n_x and n_y from (3.5) this yields two polynomials, one quartic and the other quintic in ϕ, of the form:

$$q_1\phi^4 + q_2\phi^3 + q_3\phi^2 + q_4\phi + q_5 = 0\,, \tag{3.24}$$
$$p_1\phi^5 + p_2\phi^4 + p_3\phi^3 + p_4\phi^2 + p_5\phi + p_6 = 0\,, \tag{3.25}$$

where the coefficients q_i, p_i, are real and can be expressed in terms of inner products of the flux vectors, as is discussed in appendix B.

For a non-singular solution, ϕ, must be complex. Since the coefficients of the two polynomials are real this means that (3.24), (3.25), must share a common quadratic factor.

In general, as is discussed in appendix B this happens when the following condition is met: Define the matrix

$$M \equiv \begin{pmatrix} p_1 & 0 & 0 & -q_1 & 0 & 0 & 0 \\ p_2 & p_1 & 0 & -q_2 & -q_1 & 0 & 0 \\ p_3 & p_2 & p_1 & -q_3 & -q_2 & -q_1 & 0 \\ p_4 & p_3 & p_2 & -q_4 & -q_3 & -q_2 & -q_1 \\ p_5 & p_4 & p_3 & -q_5 & -q_4 & -q_3 & -q_2 \\ p_6 & p_5 & p_4 & 0 & -q_5 & -q_4 & -q_3 \\ 0 & p_6 & p_5 & 0 & 0 & -q_5 & -q_4 \\ 0 & 0 & p_6 & 0 & 0 & 0 & -q_5 \end{pmatrix}. \tag{3.26}$$

Then (3.24), (3.25), have a common quadratic factor, if M has a zero eigenvalue. That is if a non-zero column vector X exists such that

$$M \cdot X \equiv M^i_j X^j = 0\,. \tag{3.27}$$

Note that since M is an 8×7 matrix, each column of M can be thought of as a vector in an 8 dimensional vector space. Then (3.27), is equivalent to requiring that only 6 of these 7 vectors, in the 8 dimensional space, are linearly independent.

Before proceeding let us emphasise that the matrix M depends only on the flux vectors, so the requirement of a zero eigenvalue gives rise to restrictions on the flux which must be met for a susy solution to exist.

Once condition (3.27) is met, the resulting common quadratic factor has the form

$$W(\phi) = w_1\phi^2 + w_2\phi + w_3 \tag{3.28}$$

where

$$w_1 = \frac{q_1}{s_1}\,, \qquad w_2 = \frac{s_1 q_2 - s_2 q_1}{s_1^2}\,, \qquad w_3 = \frac{q_5}{s_3}\,, \tag{3.29}$$

and s_1, s_2, s_3 refer to the first three components of X as is explained in appendix B.

We can now finally solve for ϕ by setting $W(\phi) = 0$. This gives,

$$\phi = \frac{-w_2 \pm \sqrt{w_2^2 - 4w_1 w_3}}{2w_1}\,. \tag{3.30}$$

JHEP03(2003)028

For a nonsingular solution, the imaginary part of ϕ, must not vanish. This gives the following additional conditions on the flux,

$$w_2^2 < 4w_1w_3\,. \tag{3.31}$$

Once ϕ is determined, τ can be obtained from (3.19). It is given by

$$\tau = \frac{n_x \cdot n_y}{n_x \cdot n_x}\,. \tag{3.32}$$

We should note that in case $n_x \cdot n_x = 0$, (3.32), is not valid. Instead one can use (3.16), which yields,

$$\tau = \frac{(\bar{n}_x \cdot n_y + \bar{n}_y \cdot n_x) \pm \sqrt{(\bar{n}_x \cdot n_y + \bar{n}_y \cdot n_x)^2 - 4(\bar{n}_x \cdot n_x)(\bar{n}_y \cdot n_y)}}{2(\bar{n}_x \cdot n_x)}\,. \tag{3.33}$$

(Requiring $Im(\tau) > 0$ fixes the sign ambiguity in (3.33).) eq. (3.33), is also useful for stating an additional condition on the flux which arises from the requirement that $Im\tau \neq 0$. This condition takes the form

$$(\bar{n}_x \cdot n_y + \bar{n}_y \cdot n_x)^2 < 4(\bar{n}_x \cdot n_x)(\bar{n}_y \cdot n_y)\,, \tag{3.34}$$

where ϕ, is given in (3.30).

Finally a condition on the flux arises from (3.15). Using (3.9), this takes the form,

$$(n_x\tau - n_y) \cdot (\bar{n}_x\bar{\tau} - \bar{n}_y) > 0\,, \tag{3.35}$$

with τ and ϕ given in terms of the flux, in (3.32), (3.30).

In summary, the (2,1) condition determines the complex structure of the $K3 \times T^2/Z_2$ space completely in a susy preserving solution. The dilaton-axion, ϕ, and the complex structure modulus of the T^2, τ, are given by (3.30), (3.32). The complex structure of $K3$ is determined implicitly by Ω which is given by (3.9). In addition the following conditions are imposed on the flux: the matrix M (3.26) must have a zero eigenvector, and the conditions (3.31), (3.34), and (3.35), must hold.

3.3 Primitivity

We turn next to the requirements imposed by the primitivity condition.

On $K3 \times T^2$, the Kähler two-form, J, is given by[5]

$$J = \tilde{J} + g_{z\bar{z}} dz \wedge d\bar{z}\,, \tag{3.36}$$

where $z, \bar{z}$ denote coordinates on the T^2 and $\tilde{J} \in H^{(1,1)}(K3)$.

The primitivity condition is (2.2). From the form of G_3 we see that there are no constraints on $g_{z\bar{z}}$, so the Kähler modulus of the T^2 is not fixed. The constraints on $\tilde{J}$, in terms of the inner product, (2.9), take the form,

$$\tilde{J} \cdot G_z = \tilde{J} \cdot G_{\bar{z}} = 0. \tag{3.37}$$

[5]For ease in subsequent discussion, we drop the overall factor of i, that is conventionally present in the definition of J.

Since $Im(\tau), Im(\phi) \neq 0$, for a non-singular solution, we learn from (3.5) that

$$\tilde{J} \cdot \alpha_x = \tilde{J} \cdot \alpha_y = \tilde{J} \cdot \beta_x = \tilde{J} \cdot \beta_y = 0. \tag{3.38}$$

Thus $\tilde{J}$ must be orthogonal to the vector space $V_{\rm flux}$.

An acceptable $\tilde{J}$ must meet the following two additional conditions. It should be a space-like, i.e., positive norm vector in $H^2(K3, \mathbb{R})$. And, it should be of type $(1,1)$. This latter condition can be stated as follows:

$$\tilde{J} \cdot \Omega = \tilde{J} \cdot \bar{\Omega} = 0\,. \tag{3.39}$$

From (3.9), we see that (3.39), is automatically met if (3.38), is true. This leaves the two conditions of $\tilde{J}$ being space-like and orthogonal to $V_{\rm flux}$. Since $H^2(K3, \mathbb{R})$ is 22 dimensional, at first glance, it would seem that these conditions can be met for generic fluxes, leaving 18 of the $K3$ Kähler moduli unfixed.

Some thought shows that this is not true and that in fact the flux must meet some conditions in general. The metric on $H^2(K3, \mathbb{R})$ gives rise to an inner product matrix in $V_{\rm flux}$ in an obvious manner. It is convienent to state the restrictions on the flux, in terms of the number of non-trivial eigenvectors, with positive, negative and null norm, of this matrix. One can show, as we will argue below, that the number of positive norm eigenvectors must be 2 and the number of null eigenvectors must be 0, for a non-trivial space-like $\tilde{J}$ to exist that meets, (3.38). This leaves the following three possibilities:

a) dim $V_{\rm flux} = 2$, $(2+, 0-)$

b) dim $V_{\rm flux} = 3$, $(2+, 1-)$

c) dim $V_{\rm flux} = 4$, $(2+, 2-)$,

where in our notation $(2+, 1-)$ means two eigenvectors of positive norm and one of negative norm etc.

In the next two paragraphs we pause to discuss how this conclusion comes about. Thereafter we return to the main thread of the argument again. Since Ω lies in $V_{\rm flux}$ and is space-like, the number of positive-norm eigenvectors must be at least two. Since the signature of $H^2(K3, \mathbb{R})$ is $(3, 19)$ the maximum number of positive norm eigenvectors can be three. But if it is three then $\tilde{J}$ cannot be orthogonal to $V_{\rm flux}$ and still be spacelike.[6] Thus, there must be exactly two spacelike eigenvectors in $V_{\rm flux}$.

Next, we turn to the number of eigenvectors with null norm. Any such eigenvector must be orthogonal to the two eigenvectors with positive eigenvalues. So if such an eigenvector exists, and $\tilde{J}$ is orthogonal to it, besides being orthogonal to the two space-like vectors, one can again show that it cannot be space-like.[7]

[6] One can construct a basis of $H^2(K3, \mathbb{R})$, starting from these three spacelike vectors and appending 19 time-like vectors to them. Then if $\tilde{J}$ is orthogonal to the three spacelike ones it must be purely time like.

[7] Let the two spacelike vectors be v_1, v_2 and the null vector be $v_N = v_3 + v_4$, where v_3 is spacelike and v_4 is time like. Then orthogonality would require that $\tilde{J} = v_N + v_t$, where v_t is a time like vector orthogonal to v_1, v_2, v_N. This makes $\tilde{J}$ time-like.

The argument in the previous two paragraphs shows that $V_{\rm flux}$ must meet one of the three possibilities discussed above. Once the flux meets this requirement, the condition (3.38), can be satisfied by a space-like $\tilde{J}$. Since (3.38), imposes 4 conditions this leaves 18 moduli in the $K3$ Kähler moduli space, besides the Kähler modulus of the T^2, unfixed.

Orbifold singularities. There is one final point we should discuss in this section. This is concerned with the existence of an orbifold singularity on the $K3$ surface. At such a singularity various two-cycles shrink to zero size and additional light states obtained by branes wrapping such cycles can enter the low energy theory. Since we work with a supergravity theory without these states, our analysis of the resulting vacuum is self-consistent only if it does not contain any such light state.

Usually in string theory at an orbifold point additional light states are avoided by giving an expectation value to the axionic partner of the blow-up mode of the vanishing two-cycle. In our constructions we do not always have the freedom to turn on such an expectation value, since the Kahler mode corresponding to blowing up the cycle is sometimes lifted. In such cases the axionic partner is also lifted, typically this happens because it is eaten by some Gauge Boson, in the low-energy gauged supergravity. It is an interesting question to ask what are the masses of states which arise from wrapped branes in such a case, but we have not investigated this yet.

To avoid these complications we will mainly only consider vacua below which lie away from an orbifold singularity or if at an orbifold, where the relevant blow up modes are not lifted. Two exceptions to this are the discussion towards the end of section 4.1 which describes a solution generating technique that could have wider applicability, and section 5 which discusses $\mathcal{N} = 2$ solutions. The $\mathcal{N} = 2$ vacua could prove useful in determining modifications to the BPS formulae for wrapped branes in the gauged supergravity obtained after turning on flux.

The rest of this subsection analyses how to determine if a solution contains an orbifold singularity.

One can show that the susy preserving conditions discussed above result in an orbifold singularity iff $V_{\rm flux}$ contains a Lattice Vector v of $\Gamma^{3,19}$, which is orthogonal to Ω. If no such Lattice Vector exists, one can always choose the Kähler two form $\tilde{J}$ consistent with the primitivity conditions, to avoid an orbifold.

To see this we recall from section 2 that the $K3$ surface is at an orbifold point in its moduli space if a Lattice Vector of $\Gamma^{3,19}$ exists which is orthogonal to the three plane Σ, that determines the Einstein Metric. Now if a Lattice Vector v exists which is orthogonal to Ω and $v \in V_{\rm flux}$, it must be orthogonal to $\tilde{J}$ (due to the primitivity condition (3.38)). Thus it must be orthogonal to Σ, so as claimed above the $K3$ surface is at an orbifold singularity. For the converse we need to consider two possibilities. Either there exists no Lattice Vector orthogonal to Ω, in this case we are done. Or there is such a Lattice Vector v but it does not lie in $V_{\rm flux}$. In this case one can always arrange that $\tilde{J}$, consistent with the condition, (3.38), is not orthogonal to v, so again an orbifold is avoided.[8]

[8]Decompose $v = v_{\parallel} + v_{\perp}$, where $v_{\parallel}$ lies in $V_{\rm flux}$, and $v_{\perp}$ is perpendicular to $V_{\rm flux}$. By orienting $\tilde{J}$ to have a ("small enough") component along $v_{\perp}$ one can then ensure that $\tilde{J} \cdot v \neq 0$, while (3.38), and positivity of

3.4 Summary of conditions leading to susy solutions

The discussion of this section has been quite technical and detailed. It is therefore useful to summarise the main results here for further reference.

Susy is generically broken for flux compactifications of IIB theory on $K3 \times T^2/Z_2$. To preserve susy the flux must meet the following conditions: First, the vector space $V_{\rm flux}$ spanned by the flux must be of one of the three classes, a,b,or c, discussed above in the primitivity section. Second, the matrix M, (3.26), formed from the flux, must have a zero eigenvalue.

Once these conditions are met a solution exists. The dilaton, ϕ, is given by (3.30), and in terms of ϕ the complex structure modulus of the T^2, τ, is given by (3.32). The complex structure of the $K3$ is implicitly determined by Ω which is give by (3.9) in terms of ϕ, τ. Unlike the complex structure, the Kähler moduli are not completely determined. Instead the Kähler two-form of $K3$ must meet four conditions, (3.38). This generically leaves an 18 dimensional subspace of the $K3$ Kähler moduli space, and the Kähler modulus of the T^2, unfixed.

Finally, in order to ensure that the resulting solution is non-singular some additional conditions must be met by the flux. To ensure that the complex structure moduli are stabilised at non-singular values, the inequalities, (3.35), (3.31), and (3.34), must be met. And to ensure that orbifold singularities are avoided in the resulting solution, $V_{\rm flux}$ must not contain any Lattice Vector of $\Gamma^{3,19}$, which is orthogonal to Ω.

One final comment. Our approach to finding susy preserving solutions above relied crucially on the fact that the constant c in (3.9), did not vanish. If this constant is zero the holomorphic two-form of $K3$ is not constrained to lie in $V_{\rm flux}$. This second branch of solutions will be considered further in section 5, it gives rise to $\mathcal{N} = 2$ susy preserving vacua.

4. Some examples

Here we will illustrate the general discussion of the preceeding section with a few examples. section 4.1 discusses the case where $V_{\rm flux}$ is two dimensional. We consider the general solution, some examples and also discuss a method of generating additional susy preserving solutions starting from an existing one. section 4.2 applies the general discussion above to an example where $V_{\rm flux}$ is four dimensional.

4.1 $V_{\rm flux}$ has dimension 2

In this case only two of the four flux vectors are linearly independent. Since Ω must be a two-plane contained in $V_{\rm flux}$, this means both vectors spaning $V_{\rm flux}$ must be spacelike, so that $V_{\rm flux}$ is of type $(2+, 0-)$. In this case the two-plane defined by Ω must be $V_{\rm flux}$.

Going back to the conditions for G_3 to be of type $(2, 1)$ one finds from (3.11), and (3.12), that

$$G_z \cdot \Omega = G_z \cdot \bar{\Omega} = 0 \,. \tag{4.1}$$

$\tilde{J}$ are met.

Since, in this case, $V_{\rm flux}$ is spanned by $\Omega, \bar{\Omega}$, this means from (4.1), that

$$G_z = n_x \bar{\tau} - n_y = 0\,. \tag{4.2}$$

Since $Im\tau \neq 0$ for a non-singular solution, we learn from (3.10), that

$$n_x \cdot n_x = 0\,, \tag{4.3}$$

which can be rewritten as

$$(\alpha_x \cdot \alpha_x) - 2(\alpha_x \cdot \beta_x)\phi + (\beta_x \cdot \beta_x)\phi^2 = 0\,.$$

Solving for ϕ we obtain

$$\phi = \frac{1}{(\beta_x \cdot \beta_x)}\left((\alpha_x \cdot \beta_x) \pm \sqrt{(\alpha_x \cdot \beta_x)^2 - (\alpha_x \cdot \alpha_x)(\beta_x \cdot \beta_x)}\right) \tag{4.4}$$

$\mathrm{Im}\,\phi \neq 0$ implies $(\alpha_x \cdot \beta_x)^2 < (\alpha_x \cdot \alpha_x)(\beta_x \cdot \beta_x)$. So the vectors α_x and β_x must be linearly independent.

With ϕ fixed, (4.2), is two complex equations in τ, one of these can be used to determine τ, the other then gives two real conditions on the flux. Multiplying both sides of (4.2) by $\bar{n}_x$ we have that

$$\bar{\tau} = \frac{n_y \cdot \bar{n}_x}{n_x \cdot \bar{n}_x}\,, \tag{4.5}$$

or equivalently

$$\tau = \frac{\bar{n}_y \cdot n_x}{n_x \cdot \bar{n}_x}\,. \tag{4.6}$$

Multiplying (4.2) by n_x gives

$$n_x \cdot n_y = 0\,, \tag{4.7}$$

substituting for ϕ, τ from (4.4), (4.6), in (4.7), we get the two conditions on the flux mentioned above. Finally we note that since Ω, is a spacelike two plane by construction in this case, the inequality (3.35) is automatically met. Also since Ω spans $V_{\rm flux}$, the moduli of $K3$ can be chosen to be away from an orbifold point.

Next we turn to primitivity (3.38), which requires that $\tilde{J}$ is orthogonal to the vector space $V_{\rm flux}$. In the present example, this condition does not yield any extra restrictions on the flux. It can be met easily. $H^2(K3, \mathbb{R})$ is 22 dimensional. (3.38), imposes two conditions allowing for all twenty Kähler moduli of $K3$, and the one Kähler modulus of the T^2, to vary.

As a concrete example consider the case where α_x, β_x are the two linearly independent flux vectors, with

$$\alpha_y = -\beta_x \qquad \text{and} \quad \beta_y = \alpha_x\,. \tag{4.8}$$

In addition take the flux to satisfy the conditions

$$\alpha_x^2 = \beta_x^2\,, \tag{4.9}$$

and

$$\alpha_x \cdot \beta_x = 0\,. \tag{4.10}$$

From (4.4), one finds then that the dilaton is stabilised at the value

$$\phi = \pm i\,. \tag{4.11}$$

Taking $\phi = i$ from (4.6), we have that

$$\tau = i\,. \tag{4.12}$$

We see that with this choice of flux, (4.7), is automatically met. As mentioned above in this case, Ω corresponds to the two plane spanned by α_x, β_x.

Next we come to the tadpole conditions. From (2.8), and (4.8), we see that $N_{\rm flux}$ is given by

$$N_{\rm flux} = 2\alpha_x^2\,, \tag{4.13}$$

so that the D3 brane tadpole condition takes the form

$$\alpha_x^2 + N_{D3} = 24\,. \tag{4.14}$$

This condition can be easily met.

As a specific illustration, we take, in the notation of appendix A,

$$\alpha_x = 2e_1\,, \quad \beta_x = 2e_2\,, \tag{4.15}$$

so that conditions (4.9), (4.10), are met (the even coefficients in (4.15), ensure the correct quantisation conditions). We then have that

$$N_{D3} = 24 - 8 = 16\,, \tag{4.16}$$

number of branes must be added in the compactification. Let us end by mentioning that in this example, (4.15), primitivity requires the Kähler two-form of $K3$ to be of the form

$$\tilde{J} = \sum_{i=3}^{22} t_i e_i\,,$$

where the real parameters t_i are chosen to make $J \cdot J > 0$. This is a twenty dimensional space, as was mentioned above.

New solutions from old. The case where $\dim(V_{\rm flux}) = 2$ also allows us to illustrate a trick which is sometimes helpful in finding additional solutions to the susy conditions. The idea is as follows: Given a set of flux vectors which give rise to a susy solution, one can try to alter the flux vectors in such a manner that we keep the dilaton and complex structure of both the $K3$ and T^2 unchanged. In particular this means keeping $G_{\bar{z}}$ unchanged (3.9). Let the $\alpha_x \to \alpha_x + \delta\alpha_x$ etc. Then we have that

$$\delta G_{\bar{z}} = \delta n_x \tau - \delta n_y = 0\,. \tag{4.17}$$

Since G_z must still be of type $(1,1)$ for preserving susy we have that

$$\delta G_z \cdot \Omega = \delta G_z \cdot \bar{\Omega} = 0\,. \tag{4.18}$$

This yields from (3.9), (4.17), that

$$\delta n_x \cdot \Omega = \delta n_x \cdot \bar{\Omega} = 0\,. \tag{4.19}$$

If a δn_x can be found, consistent with the quatisation conditions on the flux, (2.5), that satisfies (4.19), then (4.17) can be solved for δn_y. In some cases, as we now illustrate, δn_y is also consistent with the quantisation conditions. In this case, subject to the primitivity condition and the $D3$ brane tadpole condition (2.3) also being met, one can obtain additional susy preserving solutions.

As an example consider the set of flux, (4.8), discussed in the previous section. In this example to begin with, $dim.V_{\text{flux}} = 2$, and $\phi = \tau = i$ in the susy vacuum. Now suppose the flux is changed so that $\delta\alpha_x, \delta\beta_x$ are both orthogonal to α_x, β_x. In addition we assume that

$$\delta\alpha_y = \delta\beta_x\,, \tag{4.20}$$

and

$$\delta\beta_y = -\delta\alpha_x\,. \tag{4.21}$$

It is easy to then see that both (4.17) and (4.18) are met. For the new flux $\text{dim}V_{\text{flux}} > 2$, so the primitivity condition can also be met if $\delta\alpha_x, \delta\beta_x$ are both time like.

Let us end with two comments.

First, the change in N_{flux} is given by

$$\delta N_{\text{flux}} = -(\delta\beta_x \cdot \delta\alpha_y) + \delta\beta_y \cdot \delta\alpha_x \tag{4.22}$$

i.e.,

$$\delta N_{\text{flux}} = -\delta\beta_x^2 - \delta\alpha_x^2\,. \tag{4.23}$$

For time like $\delta\beta_x, \delta\alpha_x$ this is positive. So by altering the flux in this manner the number of D3 branes that need to be added can be reduced, (2.3). In particular one can easily find choices of $\delta\alpha_x, \delta\beta_x$ which give rise to a vacuum where $N_{\text{flux}} = 24$ and no D3-branes need be added.

Second, the above examples with $\text{dim}V_{\text{flux}} > 2$, which are generated from the old solutions by altering the flux vectors, correspond to orbifold singularities. This follows from the discussions on orbifold singularities in section 3.3 and because of the fact that the lattice vectors $\delta\alpha_x, \delta\alpha_y \in V_{\text{flux}}$ are orthogonal to Ω.

4.2 A solution with common quadratic

We now find some solutions of the quartic (3.24) and quintic (3.25). For simplicity we restrict the flux vectors to satisfy

$$(\alpha_x \cdot \alpha_y) = (\beta_x \cdot \beta_y) = (\alpha_x \cdot \beta_y + \alpha_y \cdot \beta_x) = 0\,. \tag{4.24}$$

We further assume

$$\begin{aligned}(\beta_x \cdot \beta_x) &= 2(\alpha_x \cdot \alpha_x) = 2(\alpha_x \cdot \beta_x)\\ (\beta_y \cdot \beta_y) &= 2(\alpha_y \cdot \alpha_y) = 2(\alpha_y \cdot \beta_y)\,.\end{aligned} \tag{4.25}$$

With this assumption the quartic and quintic reduces to

$$\begin{aligned}\alpha_{xx}\alpha_{yy}\left(4\phi^4 - 8\phi^3 + 8\phi^2 - 4\phi + 1\right) &= 0\,,\\ \alpha_{xx}^2\alpha_{yy}\left(-4\phi^5 + 12\phi^4 - 16\phi^3 + 12\phi^2 - 5\phi + 1\right) &= 0\,,\end{aligned} \tag{4.26}$$

where α_{xx} and α_{yy} are defined in appendix B. The above polynomials can be rewritten as

$$\begin{aligned}\alpha_{xx}\alpha_{yy}\left(2\phi^2 - 2\phi + 1\right)^2 &= 0\,,\\ \alpha_{xx}^2\alpha_{yy}(1-\phi)\left(2\phi^2 - 2\phi + 1\right)^2 &= 0\,.\end{aligned} \tag{4.27}$$

Clearly, they share a common quadratic

$$2\phi^2 - 2\phi + 1 \tag{4.28}$$

which can be set to zero to obtain solution for ϕ as

$$\phi = \frac{1}{2}(1 \pm i)\,. \tag{4.29}$$

From (3.33) and on using the assumptions (4.24), (4.25) the expression for τ becomes

$$\tau = i\sqrt{\frac{\alpha_{yy}}{\alpha_{xx}}}\,. \tag{4.30}$$

The conditions (4.24), (4.25) can be met by some suitable choice of the flux vectors. For example consider

$$\begin{aligned}\alpha_x &= 2(e_1 - e_2)\,,\\ \alpha_y &= 2(e_1 + e_2 + e_4)\,,\\ \beta_x &= -4e_2\,,\\ \beta_y &= 2(2e_1 + e_4 + e_5)\,.\end{aligned} \tag{4.31}$$

This is a solution of $(2+, 2-)$-type. For these flux vectors $\alpha_{xx} = 16$ and $\alpha_{yy} = 8$. Hence, we have $\tau = i/\sqrt{2}$. The resulting solution is non-singular. $Im(\phi), Im(\tau) \neq 0$, and one can show that (3.35), is also met. One can also show that orbifold singularities are avoided. Since $Im(\tau)$ is irrational, there is no element of $\Gamma^{3,19}$ contained in V_{flux} which is orthogonal to Ω.

Finally, we note that the contribution due to the flux to the D3-brane tadpole condition, (2.8), is given by

$$N_{\text{flux}} = \alpha_x.\beta_y - \beta_x.\alpha_y = 32\,. \tag{4.32}$$

As a result 8 $D3$ branes need to be added for a consistent solution.

5. The second branch and $\mathcal{N} = 2$ supersymmetry

As was mentioned at the very end of section 3 the general strategy discussed therein for finding susy solutions is applicable only if the constant c , (3.9), is non-zero. In section

section 5.1 we discuss how to find solutions for which this constant vanishes, by formulating both the conditions on the flux for such solutions to exist and determining the constraints on the moduli in the resulting vacua. We will refer to these solutions as lying in the "second branch". In section 5.2 we show that the second branch in fact meets the necessary and sufficient conditions for preserving $\mathcal{N} = 2$ supersymmetry. In section 5.3 we give an example of such a solution.

5.1 The second branch

We begin by noting that if c, the constant in (3.9), vanishes, then

$$G_{\bar{z}} = n_x \tau - n_y = 0\,. \tag{5.1}$$

Equating the real and imaginary parts of (5.1) separately to zero we obtain

$$\begin{aligned} \alpha_x \,\mathrm{Re}(\tau) - \beta_x \,\mathrm{Re}(\tau\phi) &= \alpha_y - \beta_y \,\mathrm{Re}(\phi)\,, \\ \alpha_x \,\mathrm{Im}(\tau) - \beta_x \,\mathrm{Im}(\tau\phi) &= -\beta_y \,\mathrm{Im}(\phi)\,. \end{aligned} \tag{5.2}$$

Thus only two of the flux vectors $\alpha_x, \alpha_y, \beta_x, \beta_y$, at best, are linearly independent. So the first thing we learn is that for a solution of this kind, dim $V_{\text{flux}} \leq 2$. Since in a nonsingular solution, $Im(\phi)$, does not vanish we can take these two independent flux vectors to be α_x, and β_x.

Next let us consider the constraints coming from primitivity. For the solutions of section 3 this was discussed in section 3.3 and much of that analysis goes over to the present case as well. In particular, one finds again that $\tilde{J}$ must be spacelike and orthogonal to V_{flux}.

The remaining constraints come from the $(2,1)$ condition. This takes the form of the following equations:

$$G_z \cdot \Omega = 0\,, \qquad G_z \cdot \bar{\Omega} = 0\,, \tag{5.3}$$

$$\Omega \cdot \Omega = 0\,, \qquad \Omega \cdot \bar{\Omega} > 0\,. \tag{5.4}$$

From (5.3) get

$$(n_x \bar{\tau} - n_y) \cdot \Omega = 0\,, \qquad (n_x \bar{\tau} - n_y) \cdot \bar{\Omega} = 0\,. \tag{5.5}$$

Using the fact, (5.1) that $n_y = \tau n_x$, and $Im(\tau) \neq 0$, the above conditions can be written as

$$n_x \cdot \Omega = 0\,, \qquad \bar{n}_x \cdot \Omega = 0$$

or equivalently

$$\alpha_x \cdot \Omega = 0\,, \qquad \beta_x \cdot \Omega = 0\ . \tag{5.6}$$

As a result we see that Ω must be orthogonal to V_{flux}.

In section 3.3 we found it useful to classify V_{flux} by the number of positive, negative, and null norm eigenvectors of the inner product matrix. In the present case, putting the constraints from the primitivity and the $(2,1)$ conditions together one can show that V_{flux} cannot contain any eigenvectors of positive, or null norm.

JHEP03(2003)028

The argument is as follows. We saw in section 2.3 that $\Omega, \tilde{J}$ together define a space-like three-plane, Σ, in $H^2(K3, R)$. Now let $v_1 \in V_{\text{flux}}$ be a positive norm eigenvector, then Ω and $\tilde{J}$ and therefore Σ, must be orthogonal to it. But since $H^2(K3, R)$ has signature $(3, 19)$, such a three-plane cannot exist. Thus a non-singular $\Omega, \tilde{J}$, requires that V_{flux} contains no positive norm eigenvector. A similar argument shows that V_{flux} cannot contain any null norm eigenvector either[9].

The only possibilities we are then left with is that V_{flux} is of dim. 2 and type $(0+, 2-)$, or of dim 1 and type $(0+, 1-)$. Once the flux meets these conditions susy preserving vacua can be found. The complex structure and Kähler two-form are somewhat constrained in these vacua but not completely fixed. Ω is orthogonal to V_{flux} and is defined by an oriented spacelike two plane in the subspace $V_\perp$ orthogonal to V_{flux}, while $\Omega, \tilde{J}$ together define a space-like three plane in $V_\perp$. E.g., in the case where dim. $V_{\text{flux}} = 2$, the space of complex structures of K3 is given (upto discrete identifications) by the grassmanian

$$G = O^+(3, 17)/(O(2) \times O(1, 17))^+ \,, \tag{5.7}$$

which is 36 dimensional, while the moduli space of Einstein metrics has the form (again upto discrete identifications),

$$M_E = O^+(3, 17)/(\mathrm{SO}(3) \times O(17)) \times R_+ \,, \tag{5.8}$$

which is 52 dimensional.

Also, while the Kähler modulus of the T^2 is not constrained. the dilaton ϕ and the complex structure of the T^2, τ, can be determined from (5.1). For example, by taking the projections of (5.1), along α_x, we find τ is given by

$$\tau = \frac{(\alpha_x \cdot n_y)}{(\alpha_x \cdot n_x)} \,. \tag{5.9}$$

Taking a projection of (5.1) along β_x and using (5.9), we then get,

$$(\beta_x \cdot n_x)(\alpha_x \cdot n_y) - (\beta_x \cdot n_y)(\alpha_x \cdot n_x) = 0 \,, \tag{5.10}$$

which is a quadratic equation in ϕ that can be solved.

To summarise, for a solution along this second branch to exist, the following conditions must be met by the flux: dim. $V_{\text{flux}} \leq 2$, and V_{flux} must be spanned by time like vectors[10]. The dilaton and τ are then determined by (5.10), (5.9). The complex structure and Kähler moduli of $K3$ are somewhat constrained but not determined completely, and the Kähler modulus of the T^2 is not constrained at all.

[9] If $v_N \in V_{\text{flux}}$ is a null eigenvector, we can write $v_N = v_1 + v_4$, where $v_1 \cdot v_1 > 0, v_4 \cdot v_4 < 0, v_1 \cdot v_4 = 0$. A basis of orthogonal vectors in $H^{(K3,R)}$ can be now constructed, $\mathcal{B} = \{v_1, v_4, v_2, v_3, v_5 \cdots v_{22}\}$, where v_1, v_2, v_3 are spacelike and the rest are timelike. Define $\hat{V}$ as the subspace spanned by the basis elements $\{v_2, v_3, v_5, \cdots v_{22}\}$. One can show that the existence of non-singular $\Omega, \tilde{J}$, requires the existence of a spacelike three-plane in $\hat{V}$. This is impossible since $\hat{V}$ has signature $(2, 18)$.

[10] There are some additional constraints on the flux which come from requiring that $Im(\phi)$ and $Im(\tau)$ are non zero, these can be deduced in a straightforward manner and we will not determine them explicitly here.

Before proceeding further, we would like to mention that all the solutions in the second branch correspond to orbifold singularities. From section 2.3 we learn that there exists an orbifold singularity when ever the space like three plane Σ is orthogonal to a lattice vector of $\Gamma^{3,19}$. Primitivity requires the flux vectors to be orthogonal to the Kähler form $\tilde{J}$. From (5.6) we find that they are also orthogonal to Ω. Hence the three plane Σ is orthogonal to $V_{\rm flux}$, resulting in orbifold singularities.

5.2 $\mathcal{N} = 2$ supersymmetry

We will now show that solutions in the second branch discussed above meet the necessary and sufficient conditions for $\mathcal{N} = 2$ supersymmetry.

These conditions were discussed for the T^6/Z_2 case in [9]. A similar analysis can be carried out for $K3 \times T^2/Z_2$ as well. Here we will skip some of the details and state the main results. The necessary and sufficient conditions for $\mathcal{N} = 2$ susy are the following: An $\mathrm{SO}(4) \times \mathrm{U}(1)$ group of rotations acts on the tangent space of $K3 \times T^2$. For preserving $\mathcal{N} = 2$ susy G_3 must transform as a $(3, 0)_{+2}$ representation under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1) \simeq \mathrm{SO}(4) \times \mathrm{U}(1)$. This means in the notation of this paper that $G_{\bar{z}}$ must vanish and G_z must transform as the anti self-dual representation of SO(4).[11]

Since $G_{\bar{z}}$ must vanish we see that all $\mathcal{N} = 2$ preserving solutions must lie in the second branch. We now show that all solutions in the second branch also meet the requirement of G_z being anti-self dual. To see this, we have to simply note that any vector belonging to $H^2(K3, R)$ which is orthogonal to both Ω and $\tilde{J}$ must be an anti-self dual two-form.[12] We saw above that G_z meets this condition in the second branch. This proves that all solutions in the second branch meet the necessary and sufficient conditions for $\mathcal{N} = 2$ supersymmetry.

A final comment. One should be able to associate more than one complex structure, which still keeps G_3 of type $(2, 1)$, with a solution of $\mathcal{N} = 2$ susy. For solutions in the second branch such an additional complex structure is given by taking $\Omega \leftrightarrow \bar{\Omega}$. This clearly changes the complex structure. And from the discussion of the second branch above it is easy to see that G_3 still continues to meet the $(2, 1)$ condition[13] (primitivity is of course still true, since $\tilde{J}$ is unchanged).

5.3 An example

For an example we consider the case where

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'} F_3 &= 2e_4 \wedge dy \,, \\ \frac{1}{(2\pi)^2\alpha'} \mathcal{H}_3 &= 2e_4 \wedge dx \,, \end{aligned} \tag{5.11}$$

where $e_4 \in \Gamma^{3,3} \subset \Gamma^{3,19}$ (see appendix A). And $e_4 \cdot e_4 = -2$, so that e_4 is a time-like vector.

[11] In [9] this representation was referred to as the self-dual representation. The discrepancy is due to an opposite choice of orientation, or equivalently opposite choice of sign for ϵ_{abcd} (notation of [9]), in the two papers. The choice in this paper agrees with the conventional one, [32] for $K3$.

[12] $H^2(K3, R)$ can be decomposed into, $H^+ + H^-$, the self-dual and anti self-dual subspace. Σ, the three plane formed by $\Omega, \tilde{J}$, is identical to H^+. Thus any vector orthogonal to $\Omega, \tilde{J}$, must be anti self-dual.

[13] G_z is orthogonal to $\bar{\Omega}$ and $G_{\bar{z}}$ vanishes.

JHEP03(2003)028

From, (2.6), (2.7), we see that $\alpha_x = 0, \beta_x = 2e_4, \alpha_y = 2e_4, \beta_y = 0$. So $V_{\rm flux}$ is one dimensional and is of type $(0, 1-)$, i.e., it is spanned by a time like vector. Thus the required conditions for a solution in the second branch are met. It is easy to see that in this simple case, ϕ, τ are not completely fixed. Rather, (5.1) imposes one condition on them

$$\phi\tau = -1\,. \tag{5.12}$$

The moduli space of complex structure of $K3$, is now the grassmanian $O(3,18)/(O(2)\times O(1,18))$ (upto discrete identifications), which is 38 dimensional and the moduli space of Einstein metrics on $K3$ has the form $O(3,18)/(O(3)\times O(18))\times R_+$, (again upto discrete identifications), which is 55 dimensional.

The flux contribution to three brane charge is $N_{\rm flux}/2 = 4$, (2.8), (2.3), so that 20 D3-branes need to be added in this case. It follows from the discussion in the previous subsection that this model has $\mathcal{N} = 2$ supersymmetry.

6. Examples with no supersymmetry

In this section we discuss some solutions which break supersymmetry. These are minima of the tree-level effective action with vanishing cosmological constant. Even though subleading effects will almost certainly destabilise them, these vacua have some interesting properties that make them worth discussing.

First, the susy breaking occurs because some D-terms are non-zero. The vanishing of the cosmological constant, at least at tree-level as was mentioned above, is tied to the no-scale structure of the model. The D-terms involved are the partners of the F-term of the volume modulus in the underlying $\mathcal{N} = 2$ theory obtained by compactification without flux [17]. And the vanishing of the cosmological constant is related to the well known fact that in no-scale models the F-term of the volume modulus[14] can be non-zero in a Poincare invariant vacuum. Second, we construct an infinite number of such vacua below, labelled by parameters which takes quantised values, all of which have the same number of D3-branes. The construction illustrates that despite the constraint imposed by the D3-brane tadpole, (2.3), the tree-level action admits an infinite number of of inequivalent fluxes and vacua. It would be very interesting to ask if there are similar families of susy preserving vacua. Finally, in this family as the flux grows "large" one finds that the complex structure is stabilised at increasingly extreme values. A similar family in the T^6/Z_2 compactification, which we briefly discuss in section 8.2, can give rise to compactifications with two large extra dimensions. This is amusing in view of recent suggestions, [33, 34].

The idea behind the construction is a follows. Consider starting with a case where the flux vectors $\alpha_x, \alpha_y, \beta_x, \beta_y$, yield a consistent susy solution. Suppose a null vector, $v \in H^2(K3, \mathbb{Z})$, $v \cdot v = 0$, exists which is orthogonal to all four of these flux vectors. Then one can consider modifying the flux vectors by adding arbitrary (even integer) multiples of the null vector v. E.g $\alpha_x \to \alpha_x + n_x v$ etc. Clearly the equations (3.13), (3.14), (3.10), still continue to hold for the same values of ϕ, τ as before and the inequality (3.15) is still

[14]This is the modulus with Kähler potential $K = -3log(-i(\rho - \bar{\rho}))$.

met. So with τ and ϕ fixed at the same value as before the $(2,1)$ condition is met for the new fluxes as well. Note that the new complex structure of $K3$ will change on modifying the flux vectors. From (3.9) we see that $G_{\bar{z}}$ and therefore Ω will be different.

Next let us consider the primitivity condition. It is easy to see that this cannot be met for the modified flux vectors. This follows from the discussion in section 3.3. The inner product matrix in V_{flux}, after the modification, will have one null eigenvector. As a result, one cannot find a spacelike Kähler two-form in $K3$ orthogonal to V_{flux}.

With the choice of complex structure moduli mentioned above, the $(2,1)$ condition is met, and we have a vacuum of the tree-level action, as was discussed in section 2.1. Since the primitivity condition is not met though, supersymmetry is broken. One can show that some D-terms do not vanish in this solution.

As a concrete example consider starting with the flux, (4.8), with $dimV_{\text{flux}} = 2$, discussed in section 4.1. A null vector v can always be found in this case orthogonal to V_{flux}. We can now modify the flux vectors as follows, $\alpha_y, \beta_x, \beta_y$ are unchanged, while,

$$\alpha_x \to \alpha_x + 2nv\,. \tag{6.1}$$

Then as argued above, the $(2,1)$ condition will be met for all values of n, and the N_{flux}, (2.3), will remain unchanged as well. One can show that the different fluxes obtained by varying the integer n are not all equivalent under U Duality. e.g. large coordinate transformations on K3 cannot turn the starting flux, to (6.1). This is because to begin, $\alpha_x = \beta_y$, but only α_x varies as n is changed. Similarly S duality which exchanges $\mathcal{H}_3$ and F_3 and T-duality on the T^2 also do not relate these different choices. As a final comment we note that in this example that for large n the complex structure of $K3$ gets stabilised at extreme values. This will became clearer in the discussion of section section 8 .

7. Duality

In this section we will study various dual descriptions of the IIB theory on $K3 \times T^2/Z_2$ in the presence of flux.

One T-duality will take us to type-IIA (or type I') with $O8$ planes. Two T-dualities will lead to a type-I description. Finally a further S-duality will give rise to a Heterotic description. The dual descriptions are not (conformally) Calabi-Yau spaces, in fact they are not even Kähler manifolds. They are related to compactifications of the Heterotic string with Torsion, [38, 22] and the more recent constructions in [23]–[25]. A general understanding of such compactifications is still not available in the literature. Our discussion will parallel that of [25], and we will use similar notation below.

The supergravity backgrounds for the three duals mentioned above can be explicitly constructed for all IIB flux compactifications on $K3 \times T^2/Z_2$. The $\mathcal{H}_3$ flux in the starting theory must have two legs along the $K3$ and one along the T^2, see (2.7). The isometries along both the directions of the T^2 can be then made manifest by choosing a gauge where the two-form NS gauge potential, B, has no dependence on the two T^2 directions. One and two T-dualities along these directions can then be explicitly carried out and the supergravity backgrounds can be obtained using [39]–[41]. Following this it is straightforward to carry out the S-duality as well.

However, since several moduli are fixed in the starting description, it is not always possible to go to a region of moduli space where the sugra description is valid in the dual theory. This problem can be avoided in cases where the moduli are only partially lifted, in such situations the dual sugra description can sometimes be reliable. An example of such a compactification for the T^6/Z_2 case was explored in [25], similar examples for $K3 \times T^2$ can also be constructed, but we will not elaborate on them here.

We will also construct a superpotential in the dual theories below. It will involve the appropriate RR and NS fluxes as well as certain "twists" in the geometry.

One comment about notation. In this section $\mu, \nu, \ldots$ denotes all the compact directions. The two directions of the T^2 are denotes by x, y. We will carry out T-duality along the x direction first and then along the y direction. $\alpha, \beta, \ldots$ will denote compact directions other than x and $\hat{\alpha}, \hat{\beta}, \ldots$ compact directions other than x and y.

7.1 One T-duality

We carry out the T-duality along the x direction of the T^2. As was mentioned above one can always choose a gauge in which $\mathcal{B}_{x\hat{\alpha}}$ is independent of the T^2 directions.

We denote the metric of IIB theory before duality by $j_{\mu\nu}$, and the metric in the IIA theory after T-duality by $g_{\mu\nu}$. The metric of the resulting manifold $\mathcal{M}'$ is given by

$$ds^2_{\mathcal{M}'} = \frac{1}{j_{xx}} \eta^x \eta^x + \frac{1}{j_{xx}} (\det{}_{xy} j)\, dy^2 + j_{\hat{\alpha}\hat{\beta}} dx^{\hat{\alpha}} dx^{\hat{\beta}} \tag{7.1}$$

where the one form $\eta^x = dx - \mathcal{B}_{x\hat{\alpha}} dx^{\hat{\alpha}}$, which can also be written as $\eta^x = g_{x\mu} dx^\mu / g_{xx}$ and $\det_{xy} j \;=\; j_{xx} j_{yy} - j_{xy}^2$. Note that $\mathcal{B}_{x\hat{\alpha}} dx^{\hat{\alpha}}$ varies non-trivially along the $K3$. As a result the resulting compactification is a sort of "twisted" analogue of the $K3 \times T^2$ space.[15] It would be quite useful to have a more complete understanding of such compactifications. In [25], it was shown that the dual compactifications could be thought of as cosets which are generalisations of the nilmanifold. It would be interesting to ask if there is a similar description in the present case.

Two more comments are in order at this stage. First, besides the metric, the RR forms F_4, F_2, and the NS form H_3, are also excited in this background. Their values can be determined using the formulae in appendix C, [40, 41], but we will not do so here. Second, one can define a two form

$$\omega_{(x)} = -d(g_{x\alpha} dx^\alpha / g_{xx})\,. \tag{7.2}$$

This is the x component of the antisymmetrised spin connection. The x direction is an isometry of the IIA metric and $\omega_{(x)}$ is the field strength of the Kaluza Klein gauge symmetry associated with this isometry. It will enter our discussion of the superpotential below.

Superpotential In writing down a superpotential in the dual theory which is the analogue of (3.1), it is first convenient to define an almost complex structure (ACS) as follows.

Define the one-form

$$\eta^z = \eta^x + i\sqrt{det_{xy} j}\, dy\,. \tag{7.3}$$

[15] It can be shown by an explicit calculation that due to the non-trivial twist, $\mathcal{M}'$ is not Ricci flat. It follows then that it cannot be a Calabi Yau manifold.

The metric (7.1) can then be written as

$$ds^2_{\mathcal{M}'} = \frac{1}{j_{xx}}\eta^z\bar{\eta}^{\bar{z}} + ds^2_{K3}\,, \tag{7.4}$$

with ds^2_{K3} denoting the metric over $K3$,

$$ds^2_{K3} = j_{\hat{\alpha}\hat{\beta}}dx^{\hat{\alpha}}dx^{\hat{\beta}}\,. \tag{7.5}$$

Consider a complex structure on $K3$ compatible with the metric, (7.5). Let dz^1, dz^2 be holomorphic one forms (in the space spanned by $dx^{\hat{\alpha}}, \hat{\alpha}, 1, \cdots 4$) with respect to this complex structure. Then the required almost complex structure we use below is defined by specifying a basis of holomorphic one forms to be η^z, dz^1, dz^2. A holomorphic $(3,0)$ form Ω_{IIA} can be constructed, it is

$$\Omega_{IIA} = \Omega \wedge \eta^z\,, \tag{7.6}$$

where $\Omega \sim dz^1 \wedge dz^2$ is the holomorphic $(2,0)$ form on $K3$. This ACS is analogous to that used in [25]. While we omit the details here, the spinor conditions take a convenient form with this choice of ACS and as a result a superpotential can also be easily constructed.

The superpotential is given by

$$W_{IIA} = \int_{\mathcal{M}'} G_{IIA} \wedge \Omega_{IIA} \tag{7.7}$$

where

$$G_{IIA} = \left(\tilde{F}_{4(x)} + g_{xx}\eta^x \wedge F_2\right) - i(\sqrt{g_{xx}}/g_s^{\rm IIA})\left(H_3 - g_{xx}\eta^x \wedge \omega_{(x)}\right). \tag{7.8}$$

Here we have used the definitions

$$\tilde{F}_4 = dC_3 + A_1 \wedge H_3\,, \qquad \left[\tilde{F}_{4(x)}\right]_{\alpha\beta\gamma} = \tilde{F}_{x\alpha\beta\gamma}\,. \tag{7.9}$$

The last term of the superpotential contains a component of the spin connection which was discussed above in (7.2). It arises from the term in the starting IIB superpotential (3.1), proportional to $\mathcal{H}_3$ with one leg along the x direction.

Evidence in support for this superpotential includes the following. First, susy requires that G_{IIA} is of type $(2,1)$ with respect to the ACS defined above. This condition is obtained by varying the superpotential (7.7). Second, the various terms in the superpotential (7.7), correctly account for the tension of various BPS domain walls in the theory. In particular the last term, involving the spin connection, gives the tension of a KK monopole related to the x isometry direction.

7.2 Two T-dualities

We can now further T-dualise along the y direction to obtain a type-I compactification with F_3 flux and twists in the geometry. We denote the dual manifold as $\mathcal{M}$. The metric on $\mathcal{M}$ is

$$ds^2_{\mathcal{M}} = \left(\hat{j}_{xx}\hat{\eta}^x\hat{\eta}^x + \hat{j}_{yy}\hat{\eta}^y\hat{\eta}^y + 2\hat{j}_{xy}\hat{\eta}^x\hat{\eta}^y\right) + ds^2_{K3} \tag{7.10}$$

with

$$\hat{\jmath}_{xx} = \frac{\jmath_{yy}}{\det_{xy}\jmath}\,, \qquad \hat{\jmath}_{yy} = \frac{\jmath_{xx}}{\det_{xy}\jmath}\,, \qquad \hat{\jmath}_{xy} = -\frac{\jmath_{xy}}{\det_{xy}\jmath}\,, \tag{7.11}$$

and

$$\hat{\eta}^x = dx + \frac{\hat{\jmath}_{yy}}{\det_{xy}\hat{\jmath}}\left(\hat{\jmath}_{xx}\hat{\jmath}_{(x)} - \hat{\jmath}_{xy}\hat{\jmath}_{(y)}\right), \tag{7.12}$$

$$\hat{\eta}^y = dy + \frac{\hat{\jmath}_{xx}}{\det_{xy}\hat{\jmath}}\left(\hat{\jmath}_{yy}\hat{\jmath}_{(y)} - \hat{\jmath}_{xy}\hat{\jmath}_{(x)}\right). \tag{7.13}$$

Note, we use '$\hat{\ }$' to denote quantities in the type-I theory.

A superpotential can be defined in this case as well. The metric (7.10) can be rewritten as

$$ds^2_{\mathcal{M}} = \hat{\jmath}_{xx}\hat{\eta}^z\hat{\eta}^{\bar{z}} + ds^2_{K3}\,. \tag{7.14}$$

An almost complex structure can be now be specified by defining one holomorphic one-form to be

$$\hat{\eta}^z = \hat{\eta}^x + \hat{\tau}\hat{\eta}^y\,,$$

and two additional holomorphic one-forms to be compatible with the complex structure of $K3$.

A $(3,0)$ form $\hat{\Omega}$ is then defined by

$$\hat{\Omega} = \Omega \wedge \hat{\eta}^z\,. \tag{7.15}$$

where Ω is the holomorphic two-form of $K3$.

The resulting superpotential is

$$\hat{W} = \int_{\mathcal{M}} \hat{G} \wedge \hat{\Omega} \tag{7.16}$$

where

$$\begin{aligned}\hat{G} = {} & \left(\hat{\jmath}_{xx}\hat{\eta}^x + \hat{\jmath}_{xy}\hat{\eta}^y\right) \wedge \hat{F}_{3(y)} - \left(\hat{\jmath}_{xy}\hat{\eta}^x + \hat{\jmath}_{yy}\hat{\eta}^y\right) \wedge \hat{F}_{3(x)} \\ & - \left(\frac{i}{g_s^I}\right)\sqrt{\det_{xy}\hat{\jmath}}\left(\hat{\jmath}_{xx}\hat{\eta}^x \wedge d\hat{\jmath}_{(x)} + \hat{\jmath}_{yy}\hat{\eta}^y \wedge d\hat{\jmath}_{(y)}\right).\end{aligned} \tag{7.17}$$

Some additional notation used in the above formula is as follows. The one and two forms $\hat{\jmath}_{(x)}$, $\hat{H}_{3(x)}$, $\hat{F}_{3(x)}$ are given as

$$\hat{\jmath}_{(x)} = \frac{1}{\hat{\jmath}_{xx}}\hat{\jmath}_{x\hat{\alpha}}dx^{\hat{\alpha}}\,, \qquad \left[\hat{F}_{3(x)}\right]_{\hat{\alpha}\hat{\beta}} = \left[\hat{F}_3\right]_{x\hat{\alpha}\hat{\beta}}\,, \tag{7.18}$$

and similar expressions for the quantities carrying the index 'y'. Also we have used the definition

$$\det_{xy}\hat{\jmath} = \left(\hat{\jmath}_{xx}\hat{\jmath}_{yy} - \hat{\jmath}^2_{xy}\right), \tag{7.19}$$

and the type-I string coupling $g_s^I = e^{\phi^I}$.[16]

[16] Note that The general expression for the superpotential in [25] after T-dualizing along both x and y appears with terms containing $\hat{\tilde{F}}_{5(xy)}$, $\hat{F}_1$ and $\hat{H}_3$ but are absent here in the superpotential (7.16) . This is due to the fact that terms containing the above quantities arise from $[F_3]_{\hat{\alpha}\hat{\beta}\hat{\gamma}}$, $[F_3]_{xy\hat{\alpha}}$, $[H_3]_{\hat{\alpha}\hat{\beta}\hat{\gamma}}$ of the original type-IIB theory. However they are projected out in $K3 \times T^2/Z_2$ compactification of IIB.

JHEP03(2003)028

7.3 Heterotic dual

Making a further S-duality we obtain heterotic theory on a manifold $\mathcal{M}_{\rm het}$ whose metric (in string frame) is given by

$$ds^2_{\rm het} = j^h_{xx}\eta^z_h\bar{\eta}^{\bar{z}}_h + g_s^{\rm het} ds^2_{K3}\,. \tag{7.20}$$

We denote the metric components in the heterotic theory with a superscript ' h ', which are related to the type I metric by a factor of the string coupling $g_s^{\rm het}$:

$$\hat{j}_{\mu\nu} = \frac{1}{g_s^{\rm het}} j^h_{\mu\nu}\,.$$

We choose an almost complex structure in the heterotic theory, which agrees with the one described above for the type-I case. The corresponding $(3,0)$ form is then given by $\hat{\Omega}$, (7.15), and the superpotential is the same as (7.16). Expressed in heterotic language this takes the form,

$$W_{\rm het} = \int_{\mathcal{M}_{\rm het}} G_{\rm het} \wedge \hat{\Omega} \tag{7.21}$$

with

$$\begin{aligned} G_{\rm het} =& \left(j^h_{xx}\hat{\eta}^x + j^h_{xy}\hat{\eta}^y\right) \wedge H_{3(y)} - \left(j^h_{xy}\hat{\eta}^x + j^h_{yy}\hat{\eta}^y\right) \wedge H_{3(x)} - \\ & - i\sqrt{\det_{xy} j^h}\left(j^h_{xx}\hat{\eta}^x \wedge dj^h_{(x)} + j^h_{yy}\hat{\eta}^y \wedge dj^h_{(y)}\right). \end{aligned} \tag{7.22}$$

The two forms $H_{3(x)}, H_{3(y)}$, above are given by

$$\left[H_{3(x)}\right]_{\hat{\alpha}\hat{\beta}} = [H_3]_{x\hat{\alpha}\hat{\beta}}\,, \qquad \left[H_{3(y)}\right]_{\hat{\alpha}\hat{\beta}} = [H_3]_{y\hat{\alpha}\hat{\beta}}\,. \tag{7.23}$$

Before closing let us note that heterotic compactifications with H_3 flux have been considered in [42, 38, 22, 35, 24]. The ACS we have defined above is not integrable in general. However, the heterotic compactifications, and also the type-I models of the precceding section, are in fact complex manifolds, as can be shown from the analysis in [38], and admit an integrable ACS. It would be interesting to ask what form the superpotential takes in terms of this complex structure.

8. The T^6/Z_2 orientifold

In this section we will discuss two aspects of flux compactifications on T^6/Z_2. section 8.1 presents general susy preserving solutions for the T^6/Z_2 compactification, in analogy with the discussion in section 3 for $K3 \times T^2/Z_2$. section 8.2 deals with a family of susy breaking solutions, similar to the one in section 6 with complex structure stabilised at extreme values for large flux.

8.1 General susy solutions

Here we discuss general susy preserving solutions in the T^6/Z_2 model. We will build on the discussion in [9]. The essential idea is similar to section 3 above. The requirements on the flux for the existence of susy solutions can be stated in terms of simultaneous solutions to two polynomial equations. Once these requirements are met the complex structure moduli can be determined in terms of the flux.

We start with the superpotential,

$$W = \int G_3 \wedge \Omega_3 \,. \tag{8.1}$$

We will use the notation of [9], below. In particular,

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'}F_3 &= a^0\alpha_0 + a^{ij}\alpha_{ij} + b_{ij}\beta^{ij} + b_0\beta^0 \\ \frac{1}{(2\pi)^2\alpha'}\mathcal{H}_3 &= c^0\alpha_0 + c^{ij}\alpha_{ij} + d_{ij}\beta^{ij} + d_0\beta^0, \end{aligned} \tag{8.2}$$

where the three forms α_{ij} and β^{ij} are defined in [9]. Susy solutions satisfy the conditions $W = \partial_i W = 0$, where i denotes all complex structure moduli. It was shown in section 4.4 of [9], that the equation $\partial_{\tau^{ij}} W = 0$ can be used to solve for τ^{ij} in terms of ϕ as

$$\left(\mathrm{cof}\left(\tau - \tilde{A}\right)\right)_{ij} = \left(\mathrm{cof}\,\tilde{A}\right)_{ij} + \tilde{B}_{ij} \tag{8.3}$$

where we define $A^0 = a^0 - \phi c^0$, $A^{ij} = a^{ij} - \phi c^{ij}$ and $\tilde{A}^{ij} = A^{ij}/A^0$ and similar definations for B_0, B_{ij} and $\tilde{B}_{ij}$. Solving (8.3) for τ^{ij} we get

$$\tau^{ij} = \tilde{A}^{ij} + \frac{(\mathrm{cof}\,\mu)^{ij}}{\sqrt{\det(\mu)}} \tag{8.4}$$

where we define μ_{ij} as

$$\mu_{ij} = \left(\mathrm{cof}\,\tilde{A}\right)_{ij} + \tilde{B}_{ij} \,. \tag{8.5}$$

Thus once we know ϕ for a given flux we can determine τ^{ij} . The conditions for ϕ are obtained from the remaining two equations $W = 0$ and $\partial_\phi W = 0$. Combining $W = 0$ and $\partial_{\tau^{ij}} W = 0$ gives a quartic for ϕ

$$B_0 \mathrm{det} A - A^0 \mathrm{det} B + (\mathrm{cof}\,A)_{ij}(\mathrm{cof}\,B)^{ij} + \frac{1}{4}(A^0 B_0 + A^{ij}B_{ij})^2 = 0\,. \tag{8.6}$$

The derivation of this equation is given in appendix B of [9]. In addition we need $\partial_\phi W$ to be zero, which combined with $W = 0$ gives

$$a^0 \det(\tau) - a^{ij}(\mathrm{cof}\,\tau)_{ij} - b_{ij}\tau^{ij} - b_0 = 0\,.$$

Elliminating τ^{ij} from above we get

$$\Delta + \frac{1}{\sqrt{\det \mu}}(\mathrm{cof}\,\mu)^{ij}\Sigma_{ij} = 0 \tag{8.7}$$

where Σ_{ij} and Δ are defined as

$$\Sigma_{ij} \equiv \frac{1}{3}a^0\left(\mathrm{cof}\,\tilde{A}\right)_{ij} + \frac{1}{3}a^0\tilde{B}_{ij} - \epsilon_{ikm}\epsilon_{jln}a^{kl}\tilde{A}^{mn} - b_{ij} \tag{8.8}$$

and

$$\Delta \equiv \tilde{A}^{ij}\Sigma_{ij} + \frac{2}{3}\tilde{A}^{ij}\tilde{B}_{ij} - a^{ij}\tilde{B}_{ij} - b_0\,. \tag{8.9}$$

We take the square of (8.7) to obtain a polynomial in ϕ:

$$\left(\Sigma_{ij}(\mathrm{cof}\,\mu)^{ij}\right)^2 - \det(\mu)\Delta^2 = 0\,. \tag{8.10}$$

The equations (8.6) and (8.10) can be rewritten as

$$\mathcal{F}_4(\phi) \equiv \hat{f}_0 + \hat{f}_1\phi + \hat{f}_2\phi^2 + \hat{f}_3\phi^3 + \hat{f}_4\phi^4 = 0 \tag{8.11}$$

and

$$\mathcal{G}_{12}(\phi) \equiv \hat{g}_0 + \hat{g}_1\phi + \hat{g}_2\phi^2 + \ldots + \hat{g}_{12}\phi^{12} = 0\,. \tag{8.12}$$

It is straightforward to obtain the coefficients $\hat{f}_i$ and $\hat{g}_i$. They are determined in terms of the integers $a^0, a^{ij}, b_0, b_{ij}\cdots$ appearing in the expressions of F_3 and $\mathcal{H}_3$ as given in (8.2). However, the expressions are quite lengthy and hence and we will not write the precise formulae for them here.

As before, in order to have a nonsingular solution, the polynomials $\mathcal{F}_4(\phi)$ and $\mathcal{G}_{12}(\phi)$ must admit a common quadratic (say $\mathcal{W}_2(\phi)$) . Thus,

$$\mathcal{F}_4(\phi) = \mathcal{F}_2(\phi)\mathcal{W}_2(\phi)\,, \qquad \mathcal{G}_{12}(\phi) = \mathcal{G}_{10}(\phi)\mathcal{W}_2(\phi) \tag{8.13}$$

where $\mathcal{F}_2(\phi)$ and $\mathcal{G}_{10}(\phi)$ are polynomials in ϕ with coefficients u_i and v_i respectively. Note that here the subscripts in $\mathcal{F}, \mathcal{G}$ and $\mathcal{W}$ denotes the degree of the polynomial.

The general solution can be stated in terms of a 15×14 matrix $\hat{\mathcal{M}}$ defined interms of the quantities $\hat{f}_i$ and $\hat{g}_i$ as

$$\hat{\mathcal{M}} = \begin{pmatrix} \hat{f}_0 & 0 & 0 & . & . & . & 0 & 0 & -\hat{g}_0 & 0 & 0 \\ \hat{f}_1 & \hat{f}_0 & 0 & . & . & . & 0 & 0 & -\hat{g}_1 & -\hat{g}_0 & 0 \\ \hat{f}_2 & \hat{f}_1 & \hat{f}_0 & . & . & . & 0 & 0 & -\hat{g}_2 & -\hat{g}_1 & -\hat{g}_0 \\ . & . & . & & & & . & . & . & . & . \\ . & . & . & & & & . & . & . & . & . \\ . & . & . & & & & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & \hat{f}_3 & \hat{f}_2 & -\hat{g}_{12} & -\hat{g}_{11} & -\hat{g}_{10} \\ 0 & 0 & 0 & . & . & . & \hat{f}_4 & \hat{f}_3 & 0 & -\hat{g}_{12} & -\hat{g}_{11} \\ 0 & 0 & 0 & . & . & . & 0 & \hat{f}_4 & 0 & 0 & -\hat{g}_{12} \end{pmatrix} \tag{8.14}$$

and a column vector $\mathcal{X}$ defined as

$$\mathcal{X} = \begin{pmatrix} & u_0 \\ & u_1 \\ & u_2 \\ & v_0 \\ & v_1 \\ & . \\ 0 & . \\ & . \\ & v_{12} \end{pmatrix} . \tag{8.15}$$

The column vector must satisfy

$$\hat{\mathcal{M}}\mathcal{X} = 0 . \tag{8.16}$$

In addition the solution obtained for ϕ must be complex. The coefficients $\hat{w}_i$ in the polynomial $\mathcal{W}_2(\phi)$ can be obtained from (8.13) in terms of u_i and v_i, which themselves are solved in terms of $\hat{f}_i$, $\hat{g}_i$ from (8.15) . They must obey

$$\hat{w}_1^2 < 4\hat{w}_0\hat{w}_2 .$$

In addition the imaginary part of τ_{ij} as given in (8.4) also must be non zero.

8.2 Susy breaking with large extra dimensions

Here we construct a one parameter family of solutions to the tree-level effective action which break supersymmetry with vanishing cosmological constant. The discussion parallels section 6, closely. We will find that for large flux the resulting T^6 has two large and four small directions. In order to use the formalism developed above, we consider tori of the form $T^4 \times T^2$ and turn on three-form flux with two legs along the T^4 and one leg along the T^2 (this is consistent with the Z_2 orientifolding). The discussion of section 6 can now be largely carried over to this case with the T^4 replacing $K3$. In particular starting with a susy solution one can again show that if a null vector exists,[17] a family of susy breaking solutions can be constructed, which preserve the $(2,1)$ condition but not the primitivity condition.

As a concrete example, analogous to that discussed towards the end of section 6, we consider the case where the three-flux takes the form

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'}F_3 &= 2e_1 \wedge dx + 2e_2 \wedge dy \\ \frac{1}{(2\pi)^2\alpha'}\mathcal{H}_3 &= -2e_2 \wedge dx + 2e_1 \wedge dy , \end{aligned} \tag{8.17}$$

where in our notation $0 \leq x, y \leq 1$ are coordinate on the T^2 and $e_1, e_2, \cdots$ are two -forms on T^4 as discussed in appendix A. In the notation of section 3, section 6, this corresponds to taking, $\alpha_x = \beta_y = 2e_1$, and, $\alpha_y = -\beta_x = 2e_2$.

[17] An inner product etc can also be defined in $H^2(T^4, R)$ as is discussed in appendix A.

It is straightforward to show that in this case a susy preserving solution exists where $\phi = \tau = i$, with τ being the complex structure of the T^2, and, where the $T^4 = T^2 \times T^2$ with the complex structure of both T^2's being stabilised at the same point in moduli space, $\tau^1 = \tau^2 = i$. The Primitivity condition can also then be easily met by a Kähler two-form,

$$J = \sum_i g_{i\bar{i}} dz^i dz^{\bar{i}}\,, \tag{8.18}$$

where $i = 1, \cdots 3$, refers to the three two-tori respectively .

Now consider the vector

$$v = e_3 + e_4\,, \tag{8.19}$$

valued in $H^2(T^4, \mathbb{Z})$ (again we refer the Reader to appendix A for definitions). It is null, i.e., $v \cdot v = 0$, and orthogonal to e_1, e_2.

We can now modify the flux vectors as follows. Keep, $\alpha_y, \beta_x, \beta_y$, the same and take

$$\alpha_x \to \alpha_x + 2nv\,. \tag{8.20}$$

The $(2,1)$ condition can then be met if $\phi = \tau = i$ and Ω (the holomorphic two-form of T^4) meets the condition

$$\Omega = (\alpha_x - \phi\beta_x)\tau - (\alpha_y - \phi\beta_y)\,. \tag{8.21}$$

The primitivity condition however cannot be satisfied. As a result susy is broken, but the tree level cosmological constant is zero.

We show in appendix A that the holomorphic two-form (8.21) corresponds to the period matrix of the T^4 given by

$$\tau = \begin{pmatrix} \frac{2i}{n} & \frac{2}{n} \\ -\frac{2}{n} & \frac{2i}{n} \end{pmatrix}. \tag{8.22}$$

It then follows that as n gets large two of the four one-cycles of the T^4 are $O(n)$ times bigger than the other two. At tree-level the Kähler two-form is unfixed and the overall size of these one-cycles in the T^4 as well as the two cycles of the T^2 are free to vary. However, one expects that corrections to the tree-level theory will lift the Kähler moduli. It is reasonable to expect that these moduli will then be stabilised so that the smaller cycles of the T^4 and both one-cycles of the T^2 have length of order one in string units. This will result in a compactification with two large extra dimensions of size $O(n)$ compared to the string scale. Taking $M_s \sim 1Tev$ we get that $n \sim 10^{14}$. The large dimensions then have size of about $1mm$, [33, 34].

Let us end by noting again, that the above discussion should be taken as being suggestive at best. Once susy is broken a controlled analysis is difficult, especially since the dilaton is stabilised at $O(1)$ values.

9. Discussion

In this paper we have discussed flux compactifications of IIB string theory. Our emphasis was on the $K3 \times T^2/Z_2$ compactification, we also discussed some aspects of the T^6/Z_2 case.

There are several open questions which remain.

JHEP03(2003)028

The $K3 \times T^2/Z_2$ compactification has $D7$-branes present in it. Our analysis did not consider the effects of exciting the gauge fields on these seven branes. It would be interesting to do so, both in the supersymmetry conditions and the resulting superpotential. We mentioned in section 2.4, that generically one expects the moduli associated with the locations of the D7-branes on the T^2 to be lifted, once flux is turned on. With gauge fields excited one expects this to continue to be true. For example, the D7-branes can acquire D5-brane charge, if the gauge field which is excited has non-trivial first Chern class. In the presence of a magnetic F_3 flux the seven branes will then experience a force along the T^2 directions.

We formulated in our discussion above the conditions which must be satisfied by the flux for susy preserving solutions to exist. It would be interesting to determine how many distinct fluxes (unrelated by duality) there are which meet these conditions. In particular, one would like to know if this number is finite or infinite and if the fluxes can be made "large" subject to the restriction of total D3-brane charge being held fixed. From a model building viewpoint, we saw in this paper that large flux leads to interesting compactifications with possibly large extra dimensions. Susy preserving fluxes of this kind would allow us to construct better controlled examples. More generally, one would like to understand how many $\mathcal{N} = 1$ vacua there are in string theory, a count of the number of distinct fluxes would be a useful first step.

The next logical step in the study of flux vacua, in continuation of [9] and this paper, would be to consider (orientifolds of) Calabi-Yau threefolds with flux. At the moment, we do not see how to directly generalise the techniques devised for $K3$ to Calabi Yau threefolds. Perhaps, the best approach might be to consider a simple case with few complex structure moduli and explicitly evaluate the superpotential.[18]

We examined some dual theories related to the flux compactifications of IIB string theory above, and saw that they are not Calabi-Yau spaces. We also obtained a superpotential in these dual descriptions. Much more can be done along this direction. For example, one would like to construct examples which cannot be related to Calabi-Yau compactifications via duality.

Finally, from the viewpoint of moduli stabilisation, the most serious limitation of these models is that the volume modulus (in the IIB description) is not stabilised. It would be illuminating to consider various additional effects which could lift this direction. Non-perturbative gauge dynamics on the world volume of D7-branes, present in the $K3 \times T^2/Z_2$ example considered above, might provide a tractable example.

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JHEP03(2003)028

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A. $\Gamma^{3,19}$ lattice

In this appendix we give some details about the integral cohomology of $K3$. Our conventions for orientation etc on $K3 \times T^2/Z_2$ are also explained. Towards the end we discuss the four-torus, T^4, this is relevant to the discussion in section 8.2.

The integral cohomology $H^2(K3, \mathbb{Z})$ has the structure of an even self-dual lattice of $\Gamma^{3,19}$ of signature $(3, 19)$. We can choose a basis $\{e_i\}$ for $\Gamma^{3,19}$ such that the inner products of the basis vectors[19]

$$g_{ij} = (e_i, e_j) = \int_{K3} e_i \wedge e_j \tag{A.1}$$

is given by the following matrix

$$g_{ij} = \begin{pmatrix} H_{3,3} & 0 & 0 \\ 0 & -\mathcal{E}_8 & 0 \\ 0 & 0 & -\mathcal{E}_8 \end{pmatrix} \tag{A.2}$$

where the matrix $H_{3,3}$ is defined as

$$H_{3,3} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{A.3}$$

and $\mathcal{E}_8$ is the Catran matrix of E_8 algebra:

$$\mathcal{E}_8 = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}. \tag{A.4}$$

The basis vectors $(e_1\ ,\ e_2\ ,\ \cdots\ e_6)$ span a subspace of $H^2(K3, \mathbb{Z})$ (which we denote as $\Gamma^{3,3}$) with the metric given by $H_{3,3}$. Note that we can make a change of basis such that $H_{3,3} = 2\eta_{3,3}$ with $\eta_{3,3} = \mathrm{diag}(1, 1, 1, -1, -1, -1)$ where as $\mathcal{E}_8$ remains the same.

Let γ_2 be an element of the integral homology $H_2(K3, \mathbb{Z})$. Integrating an arbitrary two form $\alpha_2 \in H^2(K3, \mathbb{Z})$ over γ_2 results in an integer. In particular we have

$$\int_{\gamma_2} e_i \in \mathbb{Z}\,. \tag{A.5}$$

[19]We denote the inner product of lattice vectors by a *dot* i.e. $(\alpha, \beta) = \alpha \cdot \beta$.

We turn on H_3 and F_3 fluxes over three cycles which are of the type $\gamma_2 \times \gamma_1$ where γ_2 is defined earlier and $\gamma_1 \in H_1(T^2/Z_2, \mathbb{Z})$. Integrating $e_i \wedge dx$ and $e_i \wedge dy$ over $\gamma_2 \times \gamma_1$ results in integers if γ_1 is a 'full cycle' of T^2. However if γ_1 is a 'half cycle' of T^2 (a cycle which is closed in T^2/Z_2 but not in T^2) then the result is a half integers. It was pointed out by Frey and Polchinski [10] that in order to satisfy the Dirac quantization conditions in these cases one needs to turn on fluxes due to exotic orientifold planes. As in [9], here we avoid these complications by choosing the fluxes corresponding to the lattice vectors with even coefficients in $\Gamma^{3,19}$.

It is helpful to describe the cohomology basis above in detail in the T^4/Z_2 limit of $K3$. The $E_8 \times E_8$ lattice vectors correspond to the 16 blow up modes of the orbifold. Choosing coordinates $x^i, y^i, 0 \leq x^i, y^i \leq 1, i = 1, 2$, for the T^4, a basis of Z_2 invariant two-forms is given by

$$\begin{aligned} e_1 &= \sqrt{2}\left(dx^1 \wedge dx^2 - dy^1 \wedge dy^2\right) \qquad & e_2 &= \sqrt{2}\left(dx^1 \wedge dy^2 - dx^2 \wedge dy^1\right) \\ e_3 &= \sqrt{2}\left(dx^1 \wedge dy^1 + dx^2 \wedge dy^2\right) \qquad & e_4 &= \sqrt{2}\left(dx^1 \wedge dx^2 + dy^1 \wedge dy^2\right) \\ e_5 &= \sqrt{2}\left(dx^1 \wedge dy^2 + dx^2 \wedge dy^1\right) \qquad & e_6 &= \sqrt{2}\left(dx^1 \wedge dy^1 - dx^2 \wedge dy^2\right). \end{aligned} \tag{A.6}$$

With a choice of normalization,

$$\int_{T^4/Z_2} dx^1 \wedge dx^2 \wedge dy^1 \wedge dy^2 = -\frac{1}{2}, \tag{A.7}$$

these obey the conditions,

$$\begin{aligned} (e_1, e_1) = (e_2, e_2) = (e_3, e_3) = 2 \\ (e_4, e_4) = (e_5, e_5) = (e_6, e_6) = -2, \end{aligned} \tag{A.8}$$

with all other inner products being zero. We see then that $e_1, \ldots, e_6$ form a basis for $\Gamma^{3,3}$.

For completeness let us also note that in the notation of this paper, our choice of orientation on the T^2 is given by

$$\int_{T^2} dx \wedge dy = -1. \tag{A.9}$$

Finally we discuss the T^4 case. The six one forms $e_1, \cdots e_6$ defined above, (A.6), form a basis of $H^2(T^4, \mathbb{Z})$. We can define an inner product in this vector space analogous to (A.1). Holomorphic coordinates on T^4 can be defined by

$$z^i = x^i + \tau^i_j y^j, i = 1, 2. \tag{A.10}$$

The complex structure is completely specified by the period matrix τ^i_j. The holomorphic two-form $\Omega = \lambda(dz^1 \wedge dz^2)$ (λ is a constant) can be expressed in terms of the basis $e_1, \cdots e_6$ as follows:

$$\begin{aligned} \Omega = &\frac{1}{2}(1 - det\ \tau)e_1 + \frac{1}{2}(\tau^2_2 + \tau^1_1)e_2 + \frac{1}{2}(\tau^2_1 - \tau^1_2)e_3 + \\ &+\frac{1}{2}(1 + det\ \tau)e_4 + \frac{1}{2}(\tau^2_2 - \tau^1_1)e_5 + \frac{1}{2}(\tau^2_1 + \tau^1_2)e_6. \end{aligned} \tag{A.11}$$

Now with the choice of fluxes, (8.17), (8.20), one can show that Ω, (8.21), corresponds to a period matrix,

$$\tau^i_j = \begin{pmatrix} \frac{2i}{2+n} & \frac{2}{2+n} \\ \frac{-2}{2+n} & \frac{2i}{2+n} \end{pmatrix} . \tag{A.12}$$

An $\mathrm{SL}(4,\mathbb{Z})$ transformation which takes $x^1 \to x^1 - y^2, x^2 \to x^2 - y^1$, and keeps y^1, y^2 unchanged then brings τ in the form (8.22) .

B. Solving the quartic and quintic polynomials

In this appendix we discuss in more detail the conditions leading to the two polynomials, (3.24 and (3.25), having a common quadratic factor.

The polynomials are given by

$$P(\phi) \equiv p_1\phi^5 + p_2\phi^4 + p_3\phi^3 + p_4\phi^2 + p_5\phi + p_6 = 0 \tag{B.1}$$

$$Q(\phi) \equiv q_1\phi^4 + q_2\phi^3 + q_3\phi^2 + q_4\phi + q_5 = 0 \ ,. \tag{B.2}$$

where the coeffecients p_i and q_i are

$$\begin{aligned}
p_1 &= -\gamma_{xx}\beta_{xy}^2 + \beta_{xx}\beta_{xy}\gamma_{xy} - \gamma_{yy}\beta_{xx}^2 \, , \\
p_2 &= \alpha_{xx}\beta_{xy}^2 + \beta_{xx}^2\alpha_{yy} - 2\alpha_{xy}\beta_{xy}\beta_{xx} + 4\gamma_{xx}\gamma_{yy}\beta_{xx} - \beta_{xx}\gamma_{xy}^2 \, , \\
p_3 &= -4\gamma_{xx}^2\gamma_{yy} - 2\alpha_{xx}\beta_{xx}\gamma_{yy} - 4\gamma_{xx}\beta_{xx}\alpha_{yy} + 2\gamma_{xx}\alpha_{xy}\beta_{xy} + \\
&\quad +\gamma_{xx}\gamma_{xy}^2 - \alpha_{xx}\beta_{xy}\gamma_{xy} + 3\alpha_{xy}\beta_{xx}\gamma_{xy} \, , \\
p_4 &= -2\alpha_{xy}^2\beta_{xx} + 4\gamma_{xx}^2\alpha_{yy} + 2\alpha_{xx}\beta_{xx}\alpha_{yy} + 4\alpha_{xx}\gamma_{xx}\gamma_{yy} - 4\gamma_{xx}\alpha_{xy}\gamma_{xy} \, , \\
p_5 &= -\alpha_{xx}^2\gamma_{yy} + 3\gamma_{xx}\alpha_{xy}^2 - 4\alpha_{xx}\gamma_{xx}\alpha_{yy} + \alpha_{xx}\alpha_{xy}\gamma_{xy} \, , \\
p_6 &= \alpha_{xx}^2\alpha_{yy} - \alpha_{xx}\alpha_{xy}^2 \, .
\end{aligned} \tag{B.3}$$

and

$$\begin{aligned}
q_1 &= \beta_{xy}^2 - \beta_{xx}\beta_{yy} \, , \\
q_2 &= 2\gamma_{xx}\beta_{yy} - 2\beta_{xy}\gamma_{xy} + 2\beta_{xx}\gamma_{yy} \, , \\
q_3 &= -\alpha_{yy}\beta_{xx} + 2\alpha_{xy}\beta_{xy} - \alpha_{xx}\beta_{yy} - 4\gamma_{xx}\gamma_{yy} + \gamma_{xy}^2 \, , \\
q_4 &= 2\alpha_{yy}\gamma_{xx} + 2\alpha_{xx}\gamma_{yy} - 2\alpha_{xy}\gamma_{xy} \, , \\
q_5 &= \alpha_{xy}^2 - \alpha_{xx}\alpha_{yy}
\end{aligned} \tag{B.4}$$

Here we have used the notation

$$\alpha_{ij} \equiv \alpha_i.\alpha_j \, , \qquad \beta_{ij} \equiv \beta_i.\beta_j \, , \tag{B.5}$$

with $i, j = \{x, y\}$, and

$$\gamma_{xx} \equiv \alpha_x.\beta_x \, , \qquad \gamma_{yy} \equiv \alpha_y.\beta_y \, , \qquad \text{and} \quad \gamma_{xy} \equiv \alpha_x.\beta_y + \alpha_y.\beta_x \, . \tag{B.6}$$

Assume the quadratic factor of the form

$$W(\phi) \equiv w_1\phi^2 + w_2\phi + w_3$$

is the common factor of $P(\phi)$ and $Q(\phi)$. Then

$$P(\phi) = (r_1\phi^3 + r_2\phi^2 + r_3\phi + r_4)W(\phi) \tag{B.7}$$
$$Q(\phi) = (s_1\phi^2 + s_2\phi + s_3)W(\phi) \tag{B.8}$$

for some r_i and s_i . This gives in particular

$$(s_1\phi^2 + s_2\phi + s_3)P(\phi) = (r_1\phi^3 + r_2\phi^2 + r_3\phi + r_4)Q(\phi)\,. \tag{B.9}$$

Equating the coefficients of ϕ^n from both sides we get

$$\begin{aligned}
p_1s_1 - q_1r_1 &= 0\\
(p_1s_2 + p_2s_1) - (q_1r_2 + q_2r_1) &= 0\\
(p_1s_3 + p_2s_2 + p_3s_1) - (q_1r_3 + q_2r_2 + q_3r_1) &= 0\\
(p_2s_3 + p_3s_2 + p_4s_1) - (q_1r_4 + q_2r_3 + q_3r_2 + q_4r_1) &= 0\\
(p_3s_3 + p_4s_2 + p_5s_1) - (q_2r_4 + q_3r_3 + q_4r_2 + q_5r_1) &= 0\\
(p_4s_3 + p_5s_2 + p_6s_1) - (q_3r_4 + q_4r_3 + q_5r_2) &= 0\\
(p_5s_3 + p_6s_2) - (q_4r_4 + q_5r_3) &= 0\\
p_6s_3 - q_5r_4 &= 0\,.
\end{aligned} \tag{B.10}$$

Define the matrix

$$M \equiv \begin{pmatrix}
p_1 & 0 & 0 & -q_1 & 0 & 0 & 0\\
p_2 & p_1 & 0 & -q_2 & -q_1 & 0 & 0\\
p_3 & p_2 & p_1 & -q_3 & -q_2 & -q_1 & 0\\
p_4 & p_3 & p_2 & -q_4 & -q_3 & -q_2 & -q_1\\
p_5 & p_4 & p_3 & -q_5 & -q_4 & -q_3 & -q_2\\
p_6 & p_5 & p_4 & 0 & -q_5 & -q_4 & -q_3\\
0 & p_6 & p_5 & 0 & 0 & -q_5 & -q_4\\
0 & 0 & p_6 & 0 & 0 & 0 & -q_5
\end{pmatrix}. \tag{B.11}$$

And the column vector

$$X = \begin{pmatrix} s_1\\ s_2\\ s_3\\ r_1\\ r_2\\ r_3\\ r_4 \end{pmatrix}. \tag{B.12}$$

Then (B.10), can be restated as

$$M \cdot X = 0 \tag{B.13}$$

which is eq. (3.27). Thus the condition for the two polynomials sharing a common quadratic factor is that a non-zero vector X exists satisfying (B.12).

In terms of the components of X, we find, from (B.8), by comparing powers of ϕ that $W(\phi)$ is given by (3.28), (3.29).

C. Duality maps

Here we use the notation of [25] . In particular, we define the four form field strength $\tilde{F}_4 = dC_3 + A_1 \wedge F_3$ in IIA theory and the five form field strength

$$\tilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge \mathcal{H}_3 + \frac{1}{2} \mathcal{B}_2 \wedge F_3 \,.$$

We denote the T dual direction as x and also we define the one forms

$$j_{(x)} = \frac{1}{j_{xx}} j_{x\alpha} dx^\alpha \,, \qquad g_{(x)} = \frac{1}{g_{xx}} g_{x\alpha} dx^\alpha \tag{C.1}$$

and the exterior derivative

$$\omega_{(x)} = -dg_{(x)} \,. \tag{C.2}$$

In addition $F_{n(x)}$ denotes an $n-1$ form whose components are given by:

$$[F_{n(x)}]_{i_1, \cdots i_{n-1}} = [F_n]_{x i_1 \cdots i_{n-1}} \,. \tag{C.3}$$

The Neveu-Schwarz fields transform as

$$\begin{aligned}
g_{xx} &= \frac{1}{j_{xx}} \\
g_{x\alpha} &= -\frac{\mathcal{B}_{x\alpha}}{j_{xx}} \\
g_{\alpha\beta} &= j_{\alpha\beta} - \frac{1}{j_{xx}} (j_{x\alpha} j_{x\beta} - \mathcal{B}_{x\alpha} \mathcal{B}_{x\beta}) \\
B_{x\alpha} &= -\frac{j_{x\alpha}}{j_{xx}} \\
B_{\alpha\beta} &= \mathcal{B}_{\alpha\beta} - \frac{1}{j_{xx}} (j_{x\alpha} \mathcal{B}_{x\beta} - \mathcal{B}_{x\alpha} j_{x\beta}) \\
g_s^{IIA} &= \frac{g_s^{IIB}}{\sqrt{j_{xx}}} \,.
\end{aligned} \tag{C.4}$$

Here the left hand refers to fields in the IIA theory and the right hand side to fields in the IIB theory. For the three form field strength, H_3 this takes the form,

$$\begin{aligned}
H_{3(x)} &= dj_{(x)} \\
H_3 &= \mathcal{H}_3 - \mathcal{H}_{3(x)} \wedge j_{(x)} - \mathcal{B}_{(x)} \wedge dj_{(x)}
\end{aligned} \tag{C.5}$$

The Ramond fields transform as

$$\begin{aligned}
F_{2(x)} &= F_1 \\
F_2 &= \tilde{F}_{3(x)} - \mathcal{B}_{(x)} \wedge F_1 \\
\tilde{F}_{4(x)} &= \tilde{F}_3 - j_{(x)} \wedge \tilde{F}_{3(x)} \\
\tilde{F}_4 &= \tilde{F}_{5(x)} - \mathcal{B}_{(x)} \wedge \left(\tilde{F}_3 - j_{(x)} \wedge \tilde{F}_{3(x)} \right)
\end{aligned} \tag{C.6}$$

In the formulae above, a field strength with and without a leg along the x direction are denoted as $F_{n(x)}$, and F_n respectively.

The inverse of these expressions is given by

$$
\begin{aligned}
\jmath_{xx} &= \frac{1}{g_{xx}} \\
\jmath_{x\alpha} &= -\frac{B_{x\alpha}}{g_{xx}} \\
\jmath_{\alpha\beta} &= g_{\alpha\beta} - \frac{1}{g_{xx}}(g_{x\alpha}g_{x\beta} - B_{x\alpha}B_{x\beta}) \\
\mathcal{B}_{x\alpha} &= -\frac{g_{x\alpha}}{g_{xx}} \\
\mathcal{B}_{\alpha\beta} &= B_{\alpha\beta} - \frac{1}{g_{xx}}(g_{x\alpha}B_{x\beta} - B_{x\alpha}g_{x\beta}) \\
g_s^{IIB} &= \frac{g_s^{IIA}}{\sqrt{g_{xx}}} \qquad \text{(C.7)} \\
\mathcal{H}_{3(x)} &= -\omega_{(x)} \\
\mathcal{H}_3 &= H_3 + \omega_{(x)} \wedge B_{(x)} - g_{(x)} \wedge H_{3(x)} \qquad \text{(C.8)}
\end{aligned}
$$

and

$$
\begin{aligned}
F_1 &= F_{2(x)} \\
\tilde{F}_{3(x)} &= F_2 - g_x \wedge F_{2(x)} \\
\tilde{F}_3 &= \tilde{F}_{4(x)} - B_{(x)} \wedge \big(F_2 - g_x \wedge F_{2(x)}\big) \\
\tilde{F}_{5(x)} &= \tilde{F}_4 - g_{(x)} \wedge \tilde{F}_{4(x)} \qquad \text{(C.9)}
\end{aligned}
$$

The type-I theory is equivalent to the heterotic theory by a S duality under which the fields are related as

$$
\begin{aligned}
g_s^{\mathrm{het}} &= \frac{1}{g_s^I} \\
\jmath^h_{\mu\nu} &= \frac{1}{g_s^I}\hat{\jmath}_{\mu\nu} \\
H_3 &= \hat{F}_3\,. \qquad \text{(C.10)}
\end{aligned}
$$

Here we denoted the metric in heterotic theory as $\jmath^h_{\mu\nu}$ and the NS field strength $H_3 = dB_2$. g_s^{het} heterotic string coupling. Similarly, we denoted the type-I metric and RR field strength with a '$\hat{\ }$' where as the string coupling is denoted by a superscript I.

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JHEP03(2003)028

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